



# Strength of depth effects induced by three types of vertical disparity

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## Abstract

The goal of the present study is to compare the strengths of depth effects induced by different types of vertical disparity. We use a nulling task, in which the depth effects induced by vertical disparity are nulled by horizontal disparity. The advantage of this method is that it prevents cue conflicts from arising between disparity and other depth cues. The ratios between horizontal and vertical disparity that evoke the percept of a fronto-parallel stimulus vary per type of vertical disparity. The ratios determined for vertical scale and vertical quadratic mix (vertical scale with a horizontal gradient) vary strongly across subjects. The ratios for vertical shear are constant, since all subjects needed the same amount of horizontal and vertical shear to perceive a fronto-parallel plane. In these experiments, one conflict cannot be avoided, namely the conflict between vertical disparity and oculomotor signals. This conflict may cause differential weighting of vertical disparity and oculomotor signals, which could explain the individual differences. The different ratios for different types of vertical disparity suggest that weighting is specific for each type of vertical disparity and the associated oculomotor signal. © 2000 Published by Elsevier Science Ltd.

*Keywords:* Stereopsis; Binocular vision; Vertical disparity; Depth perception

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## 1. Introduction

Many studies have investigated the strength of depth effects induced by different types of vertical disparity (Ogle, 1938, 1939; Gillam, Chambers, & Lawergren, 1988; Gillam & Rogers, 1991; Rogers, 1992; Howard & Kaneko, 1994; Rogers & Bradshaw, 1995; Kaneko & Howard, 1996; Adams et al., 1996). In all these studies, the results differ across subjects. The investigators usually studied only one type of vertical disparity and the studies involved different subjects. Therefore, it is not known whether there is a relation between the strengths of depth effects induced by different types of vertical disparities. The goal of the present experiments is to compare the strength of the depth effects induced by three different types of vertical disparity fields. In order to achieve this goal, we used the same subjects, set-up and method for each type of vertical disparity field.

Current work has concentrated on three global vertical fields. Therefore, we investigated three different

types of vertical transformations that induce three different types of vertical disparity, namely vertical scale, vertical shear and vertical quadratic mix (vertical scale with a horizontal gradient, see Appendix for further information).

Following Ogle (1938, 1939), Backus and Banks (1999) and Van Ee, Banks, and Backus (1999), we used a nulling method to measure the strength of three types of global vertical headcentric disparity fields. In the literature, different nulling methods have been used to measure the strength of depth effects induced by vertical disparity. In one type of experiments, the depth effects induced by vertical disparity were nulled by all other depth cues (including horizontal disparity; Ogle, 1938, 1939; Amigo, 1972). In other experiments, the depth effects induced by vertical disparity were nulled by horizontal disparity (Stenton, Frisby, & Mayhew, 1984; Rogers & Bradshaw, 1995; Adams et al., 1996; Backus, Banks, Van Ee, & Crowell, 1999). We used the second method, in which the strength of depth effects evoked by vertical disparity was indicated by the amount of horizontal disparity needed to null the depth effects. The advantage of nulling by horizontal dispar-

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ity is that the method prevents cue conflicts from arising between disparity and other depth cues. The rationale is as follows. In the experiments, we used stereograms projected on a fronto-parallel screen. When not transformed, the stereogram is perceived as being flat, fronto-parallel plane. The stereogram is also perceived as flat and in the fronto-parallel plane when the horizontal disparity nulls the depth effect elicited by vertical disparity. Other depth cues like perspective, illuminance, blur and accommodation also indicate that the stimulus is flat and fronto-parallel. Therefore, cue conflicts between disparity and these other depth cues are absent when the depth effects induced by vertical and horizontal disparity cancel each other out.

The main difference between our experiments and the above-mentioned nulling experiments is that we investigated three different types of vertical disparity, whereas only one type of vertical disparity was investigated in the other experiments. Another important difference is that we used a forced-choice task and we determined psychometric curves. This enabled us to determine the strength of the depth effects induced by the different types of vertical disparity fields as well as the sensitivity of the subject to vertical disparity.

## 2. Methods

### 2.1. Subjects

Four subjects (aged 18–47 years) participated in the experiments. All had normal or corrected-to-normal visual acuity and normal stereoscopic vision. Two of them had experience in psychophysical experiments involving stereoscopic vision (CE, HW) and two subjects were naive (LW, PE).

### 2.2. Apparatus

An anaglyph set-up was used to generate the stereograms (see also Van Ee & Erkelens, 1996). The stimuli were produced by an HP750 graphics computer (frequency = 70 Hz) and back-projected on a fronto-parallel translucent screen by a projection TV (Barco Data 800). The subject was seated 1.50 m from the screen. The left-eye image was projected in red light and the right-eye image was projected in green light. The subject wore glasses consisting of a red filter in front of the left eye and a green filter in front of the right eye. The transmission spectra of the filters (Schott Tiel, The Netherlands) were chosen to correspond as closely as possible to the emission spectra of the projection TV. The measurements were performed in a completely dark room. A random dot pattern of 2500 dots was generated in an array of  $900 \times 900$  pixels. The resolution (the smallest change in disparity possible)

was 3.8 min of arc. The dots were always circular so that the subject would not use perspective information. We changed their positions without changing their shapes. A small dot size (diameter of 15.3 min of arc) was chosen to prevent the shape of the dots from influencing the percept as much as possible. The dots were not anti-aliased. The visual angle of the pattern was  $53 \times 53$  degrees. The sparse random dot pattern was shaped as a jittered square.

During all experiments, the head of the subject was fixated by a chin-rest. No fixation point was provided, so subjects made eye movements as they performed the task.

### 2.3. Procedure for Expt SCALE

Two half-images, which are oppositely scaled in vertical directions, are perceived as slanted about the vertical axis. Half-images which are horizontally scaled, relative to each other also induce slant about the vertical axis. In Expt SCALE, we presented different horizontal scales in combination with a specific vertical scale. In a forced-choice task, we determined the horizontal scale required to null the depth induced by the vertical scale. We repeated the experiment for five magnitudes of vertical scale (two in each direction and zero):  $-0.06$ ,  $-0.03$ ,  $0$ ,  $0.03$ , and  $0.06$ . We used factors to express the magnitudes of all transformations. For vertical scale these factors are equivalent to percentages of magnification of:  $-6\%$ ,  $-3\%$ ,  $0\%$ ,  $3\%$  and  $6\%$ . If a subject was unable to fuse vertical scaled stereograms with one of these magnitudes, we used smaller vertical scale factors. In two cases (for subjects CE and PE), the vertical scale factors were reduced to:  $-0.04$ ,  $-0.02$ ,  $0$ ,  $0.02$ , and  $0.04$ .

Scale, shear and quadratic mix are expressed in dimensionless quantities instead of percentages or degrees (see Appendix A). The use of these quantities allows us to draw comparisons between types of vertical disparity.

In order to determine the magnitude of horizontal scale needed to null the slant induced by vertical scale, we used the method of constant stimuli in a forced-choice task. The forced-choice question put to the subject was: "Which is nearer to you — the left side or the right side of the surface?" In pilot experiments, we explored for each magnitude of the vertical scale the range of horizontal scales in which the subject gave inconsistent answers (less than 100% left or right). The relevant range was divided into seven equidistant magnitudes of horizontal scale. Combinations of horizontal and vertical scale were presented in random order distributed over four sessions. Each combination was measured eight times. Psychometric curves (cumulative normal) were fitted to the data. We obtained two fit parameters: the subjective equality,  $\mu$  and the discrimi-

nation threshold,  $\sigma$ . The subjective equality  $\mu$  is the amount of horizontal scale needed to null the effect of the vertical scale. The discrimination threshold  $\sigma$  is the slope parameter. It indicates how well a subject can distinguish between surfaces slanted to the left and surfaces slanted to the right. We estimated the errors in  $\mu$  and  $\sigma$  by performing Monte-Carlo simulations on the data sets.

Each stimulus, a specific combination of horizontal and vertical scale, was presented for 30 s whereupon a response-screen appeared. The subjects made their judgements by clicking with the computer mouse the word 'left' or 'right'.

#### 2.4. Procedure for Expt SHEAR

In Expt SHEAR, we measured the magnitude of the horizontal shear needed to null the slant evoked by a certain vertical shear. Horizontal shear is used to measure the strength of the depth effect induced by vertical shear, because horizontal and vertical shear evoke the same type of depth effect, namely a slant about the horizontal axis. The procedure for Expt SHEAR is the same as the procedure for Expt SCALE. Five magnitudes of vertical shear were measured:  $-0.04$ ,  $-0.02$ ,  $0$ ,  $0.02$ , and  $0.04$ . The forced-choice question put to the subject was: "Which is nearer to you — the upper part or the lower part of the surface?"

#### 2.5. Procedure for Expt MIX

A vertical quadratic mix transformation and a horizontal quadratic scale transformation (Appendix A) are both perceived as a vertical cylinder. In Expt MIX different amounts of horizontal quadratic scale were added to a specific vertical quadratic mix. We determined the strength of the horizontal quadratic scale transformation required to null the depth induced by a vertical quadratic mix transformation. Five magnitudes of vertical quadratic mix were measured:  $-0.16$ ,  $-0.08$ ,  $0$ ,  $0.08$ , and  $0.16$ . The forced-choice question put to the subject was: "Is the surface convex or concave?" The rest of the procedure is the same as described in the procedure for Expt SCALE.

#### 2.6. Procedure for Expt TIME

Expt TIME was conducted to investigate whether the results of the Expts MIX, SCALE and SHEAR depended on the presentation time. Van Ee and Erkelens (1996) showed that slant induced by horizontal scale or shear develops to a stable level in about 10 s. Allison, Howard, Rogers, and Bridge (1998) found that both horizontal and vertical scale and shear build up with the same speed during 30 s. To examine the time characteristics of depth induced by vertical and hori-

zontal disparity, Expt MIX was repeated with only one vertical quadratic mix factor (0.08) and different presentation times (5 s, 10 s, 15 s, 20 s, 30 s, 40 s). Stimuli with different presentation times were presented in random order and distributed over three sessions.

### 3. Results

#### 3.1. Expt SCALE

The results of Expt SCALE are depicted in Fig. 1. The amount of horizontal scale needed to null the slant induced by the applied vertical scale (the  $\mu$  value) is plotted against the vertical scale for each subject. The magnitudes of the estimated errors in  $\mu$  are generally small relative to  $\mu$ , which means that the psychometric curves fitted well to the data. A linear relation (least squares) was fitted to the  $\mu$  values for each subject (Table 1). The slopes fit very well ( $R^2 = 0.93$ ). The fitted slopes differ strongly between the four subjects. The slopes are 0.5, 0.6, 0.9 and 1.2, respectively (see Table 1). The offsets do not differ significantly from zero ( $P > 0.05$ ). Thus, for each subject there is a specific ratio between horizontal and vertical scale which is perceived as fronto-parallel.

The  $\sigma$  values indicate the sensitivity of the subjects to slant judgements. They are a measure of the differences in scales for which subjects can distinguish between slants to the left or the right. Fig. 4 shows the  $\sigma$  value of each subject averaged over the  $\sigma$ 's measured at the five vertical scales. The  $\sigma$ 's are scattered widely, especially for subject PE. There is a high correlation between the magnitude of  $\sigma$  and the nulling ratios ( $R^2 = 0.86$ ).

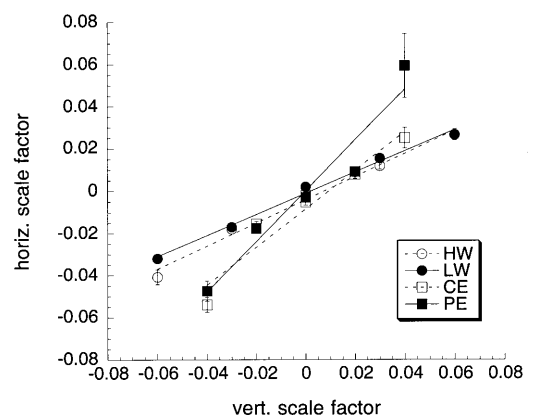


Fig. 1. The results of Expt SCALE. The  $\mu$  values are plotted against the applied vertical scale. The  $\mu$  values are the amounts of horizontal scale needed to null the effect of the vertical scale used. Different symbols represent different subjects. The errors ( $\pm 1$  SD) are estimated by a Monte-Carlo simulation. Sometimes the error bars are too small to be visible. The lines represent the least square fits for the different subjects. Note that the slopes differ strongly between subjects.

Table 1  
The linear fit parameters for Expt SCALE, SHEAR and MIX. SE means standard error<sup>1</sup>

		HW	LW	CE	PE
Expt SCALE	Slope	0.55*	0.50*	0.91*	1.20*
	SE slope	0.11	0.10	0.14	0.18
	Offset	-0.004	-0.001	-0.008	0.000
	SE offset	0.005	0.004	0.004	0.005
	$R^2$	0.99	0.99	0.93	0.93
Expt SHEAR	Slope	-0.94	-0.89	-1.08	-1.12*
	SE slope	0.03	0.05	0.08	0.03
	Offset	0.001	0.001	-0.002	0.001
	SE offset	0.001	0.001	0.002	0.005
	$R^2$	0.999	0.995	0.99	0.998
Expt MIX	Slope	0.31*	0.45*	1.15	0.83
	SE slope	0.05	0.06	0.08	0.07
	Offset	0.001	0.029*	-0.006	0.013
	SE offset	0.006	0.007	0.010	0.008
	$R^2$	0.93	0.95	0.98	0.98

<sup>1</sup> The offset marked with an asterisk differs significantly from zero ( $P < 0.05$ ). The slopes marked with an asterisk differ significantly from the slope expected from the headcentric model ( $P < 0.05$ ).

### 3.2. Expt SHEAR

Fig. 2 shows the amount of horizontal shear needed to null the slant induced by a specific vertical shear (the  $\mu$  value) for each subject and different vertical shears. The amount of shear is expressed in shear factor, which is equal to the tangent of the shear angle. The estimated errors in  $\mu$  are small relative to  $\mu$ . We carried out a linear regression (least squares) on the  $\mu$  values for each subject. The slopes fit very well ( $R^2 \geq 0.99$ ) and they are all about  $-1$  (see Table 1). Only one slope (PE) differs significantly from  $-1$  ( $P < 0.05$ ). The offsets do not differ significantly from zero ( $P > 0.05$ ).

The  $\sigma$  values indicate the differences in shears for which subjects can distinguish between upward- and downward-slanted surfaces. Fig. 4 shows a positive correlation between average  $\sigma$ 's in Expt SCALE and average  $\sigma$ 's in Expt SHEAR. In Expt SHEAR, there is no correlation between the nulling ratios and magnitude of the average sigmas, because the ratios are all about  $-1$ .

### 3.3. Expt MIX

Fig. 3 shows the results of Expt MIX. The magnitude of  $\mu$ , which is the amount of horizontal quadratic scale that nulls the effects of a particular vertical quadratic mix, is plotted against the vertical quadratic scale for each subject. The magnitudes of the estimated errors in  $\mu$  are mostly small relative to  $\mu$ . In a few cases (CE vertical quadratic mix factor,  $vqm = 0.08$ , LW  $vqm = 0.16$ , PE  $vqm = 0.16$ ) the psychometric curves did not fit well (estimated errors in  $\mu$  and  $\sigma > 0.02$ ).

We fitted a linear relation (least squares) to the  $\mu$  values for each subject (Table 1). The slopes fit very well ( $R^2 > 0.93$ ). The slopes differ strongly between subjects, namely between 0.31 and 1.15. The offset is significantly different from zero ( $P > 0.05$ ) for only one subject (LW). Apparently, for this subject perceived flatness is not the same as real flatness.

The  $\sigma$  values indicate how well subjects can distinguish between convex and concave surfaces. Fig. 4 shows that subjects, who are better at distinguishing between directions of slants, are also better at distinguishing between convex and concave surfaces. Thus,

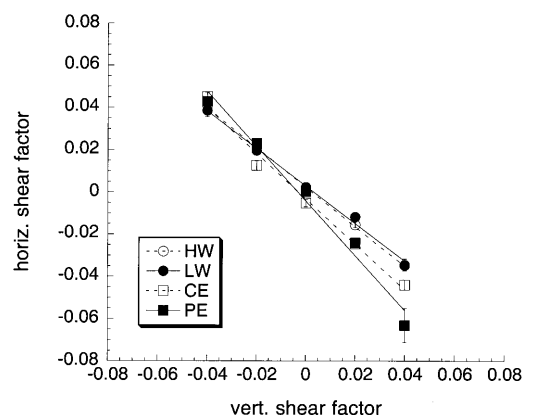


Fig. 2. The results of Expt SHEAR. The  $\mu$  values are plotted against the applied vertical shear. The  $\mu$  values are the amounts of horizontal shear needed to null the effect of the vertical shear applied. Different symbols represent different subjects. The errors ( $\pm 1$  SD) are estimated by a Monte-Carlo simulation. Sometimes the error bars are too small to be visible. The lines represent the least square fits for the different subjects. Note that the slopes are very similar for all subjects.

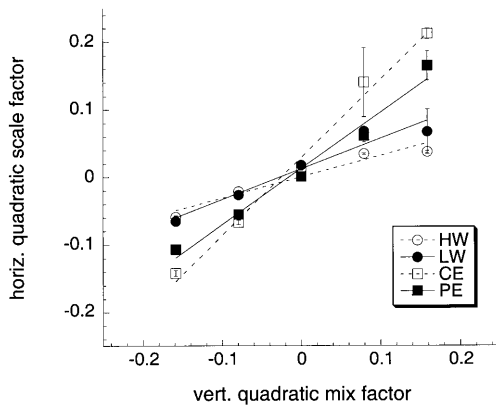


Fig. 3. The results of Expt MIX. The  $\mu$  values are plotted against the vertical quadratic mix used. The  $\mu$  values are the amounts of horizontal quadratic scale needed to null the effect of the applied vertical quadratic mix. Different symbols represent different subjects. The errors ( $\pm 1$  SD) are estimated by a Monte-Carlo simulation. Sometimes the error bars are too small to be visible. The lines represent the least square fits for the different subjects. Note that the slopes differ strongly between subjects.

some subjects are more sensitive than others to changes in disparity. The correlation between the nulling ratios and the sigmas is not significant ( $P > 0.05$ ) in Expt MIX.

The quadratic mix transformation was not always perceived as a vertical cylinder. Subjects whose results revealed a low ratio for quadratic mix (HW and LW) reported that they saw a sort of convex or concave mountain instead of a vertical cylinder. This mountain seemed to be more curved in the horizontal direction than in the vertical direction.

### 3.4. Expt TIME

In this experiment, the vertical quadratic mix factor was  $-0.08$ . The amount of horizontal quadratic scale necessary to null the effects of the vertical quadratic mix ( $\mu$ ) was determined for six different presentation

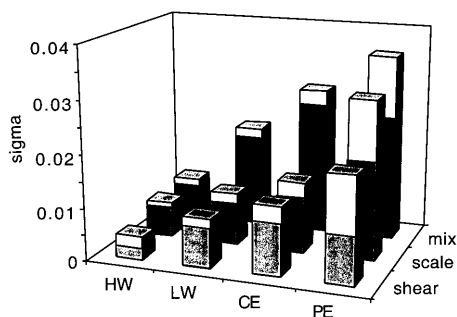


Fig. 4. The average of  $\sigma$  for each type of vertical disparity and each subject. The grey bars represent the averages over five  $\sigma$  values, namely the  $\sigma$  values of the five different amounts of a specific type of vertical disparity for one subject. The white bars on top of the grey bars represent the standard errors of the averages.

times. We fitted a linear relation (least squares) to the  $\mu$  values. The slopes did not differ significantly from zero ( $P > 0.05$ ). Thus, time did not have a significant effect.

## 4. Discussion

We quantified the depth induced by three types of vertical disparity. We compared the strength of these depth effects by measuring the amount of horizontal disparity needed to null the effects of the vertical disparity fields. Nulling ratios were determined for each subject and each type of vertical disparity field (Table 1). The nulling ratio is defined as the ratio between horizontal and vertical disparity, for which the stimulus is perceived as flat and fronto-parallel. The ratios varied slightly for the shear transformations. Only one shear ratio differed significantly from  $-1$  ( $P > 0.05$ ). By contrast, the ratios differed strongly between subjects for scale and quadratic mix transformations (a factor 2.4 for scale and 3.7 for quadratic mix). The main conclusion to be drawn from our study is that the ratios differed per type of vertical disparity and across subjects.

The sensitivity of the subject to vertical disparity is shown in Fig. 4. It seems that some subjects are more sensitive to vertical disparity than others. This effect is almost significant (two-factor ANOVA with replication,  $P = 0.052$ ).

### 4.1. Past and present results

The shear ratios were about  $-1$  in all four subjects (Table 1). A ratio of  $-1$  implies that subjects judged pure rotations of the half-images (also called curl transformations) as a fronto-parallel plane. This agrees with the findings of Howard and Kaneko (1994) and it partly agrees with the findings of Van Ee and Erkelens (1996). Four of their subjects also perceived a rotation as a fronto-parallel plane, whereas two other subjects perceived a rotation as a slanted surface. Gillam and Rogers (1991), on the other hand, found different results. They found that curl evoked a somewhat smaller slant than that induced by horizontal shear alone, which implies a ratio that is closer to zero than to one. The differences between Gillam and Rogers's results on the one hand and Howard and Kaneko's and our results on the other hand may be explained by the fact that Gillam and Rogers used a small screen (visual angle  $10^\circ$ ), which was probably too small to measure a global vertical field. Furthermore, a reference was visible. Van Ee and Erkelens (1995) showed that in the presence of a visual reference, slant perception is based solely on horizontal shear and horizontal scale transformations.

We found that the scale ratios varied from 0.5 and 1.2 (Table 1) and those magnitudes are consistent with previous observations in the literature. Stenton et al. (1984) and Kaneko and Howard (1996) found average ratios that were a little smaller than one and Backus et al. (1999) reported ratios of 0.64–0.89.

Rogers and Bradshaw (1995) studied the depth induced by the vertical quadratic mix field. They presented combinations of vertical quadratic mix and horizontal quadratic scale on the screen. The subject had to alter the amount of horizontal quadratic scale until the stimulus looked flat. We converted their results into nulling ratios. The ratios determined from Rogers and Bradshaw's experiments are 0.59, 0.62 and 0.68 for the different subjects (screen distance 57 cm, viewing angle 70°). These ratios are within the range we determined (between 0.31 and 1.15, Table 1). Adams et al. (1996) also studied the depth effects induced by the vertical quadratic mix field. The depth effects they found were comparable in size to those found by Rogers and Bradshaw (1995) in their experiments.

In conclusion, our results agree with the results reported by various authors.

#### 4.2. *Weighting of vertical disparity and oculomotor signals in depth estimation*

Underestimation or overestimation of depth has been attributed to conflicts between disparity and other depth cues (Johnston, Cumming, & Landy, 1991; Johnston, 1993). Generally, subjects estimate depth induced by disparity alone non-veridically, because different cues signal different depths. Individual differences are explained by assuming that different subjects attach different levels of reliability to different cues.

Our experimental method prevented conflicts from arising between disparity and other depth cues, like perspective, illuminance, blur or accommodation. Still, there were two disparity-based depth estimates that were not in agreement with each other in our experiments, namely depth derived from horizontal disparity scaled by vertical disparity and depth from horizontal disparity scaled by eye position. We suggest that both vertical disparity and eye position signals are used differently by the subjects causing the observed individual differences for each type of vertical disparity field. This idea of weighting agrees with the findings of Gillam et al. (1988); Backus and Banks (1999) and Backus et al. (1999) for scale and the findings of Rogers and Bradshaw (1995) for the mixed transformation. However, Gillam, Chambers and Lawergren did not carry out a nulling experiment.

The fact that we found hardly any individual differences in Expt SHEAR suggests that for this vertical disparity field weighting of the depth estimates based on vertical disparity and oculomotor signals is identical

for all subjects. The fact that the ratio is equal to  $-1$  indicates that only vertical disparity is used. Cyclovergence signals are the type of oculomotor signals that are related to vertical shear. Thus, the visual system may consider cyclovergence signals to be very unreliable, whereas it considers vertical disparity to be very reliable. It is possible that cyclovergence signals are not used in depth perception at all.

In conclusion, we suggest that the present results are due to weighting of vertical disparity and eye position signals. The fact that the ratios depend on the type of vertical disparity indicates that the visual system weights these signals differently for each type of vertical disparity.

#### 4.3. *Weighting in different models*

Weighting of different depth cues is described by the weak fusion model (Landy, Malony, Johnston, & Young, 1995) and more specifically by the estimator reliability model (Backus & Banks, 1999; Backus et al., 1999). In the estimator reliability model, the depth derived from horizontal and vertical retinal disparity and the depth derived from the eye position signals in combination with horizontal retinal disparity are weighted. So far the estimator reliability model has been applied to stereoscopic slant about the vertical axis. It predicts the results of Expt SCALE well. If this model is applied to slants about the horizontal axis and curvature, it should predict the results of Expt SHEAR and Expt MIX too. The weak fusion model does not demonstrate clearly how depth is derived from stereo, but it can explain all of our results.

According to the weak fusion model and the estimator reliability model, weighting occurs at the level of depth maps. However, a model in which weighting occurs at the level of disparity and oculomotor signals may also predict the present results. Weighting at an earlier stage than weighting of depth maps is often called interaction between depth cues. This type of weighting is described by the strong fusion model (Landy et al., 1995) and also by the regional disparity correction model (Gårding, Porrill, Mayhew, & Frisby, 1995). The RDC model combines pictorial cues, oculomotor cues and vertical disparity in both stages of the model, namely disparity correction and disparity normalisation.

The estimator reliability model is based on oculocentric co-ordinates and thus on retinal disparities, but a model based on headcentric co-ordinates may also use weighting of disparity and oculomotor signals. We propose a modification of the headcentric model (Erkelens & Van Ee, 1998). In the original headcentric model, it is assumed that retinal signals are more accurate than oculomotor signals. The model uses vertical disparity as an error signal. If an error in the oculomotor signals

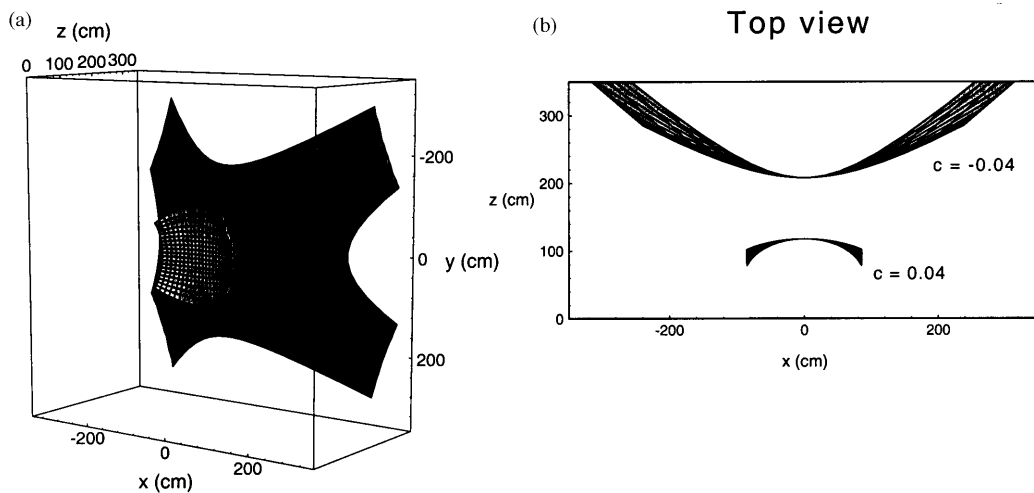


Fig. 5. (A) The percept, according to the model of Erkelens and Van Ee (1998), when a vertical quadratic mix transformation (factor 0.04 or  $-0.04$ ) is presented to a subject who has a weighting factor of 0.3. The model assumes that depth estimation by horizontal disparity is vertical. The subject is positioned with his cyclopean eye at the origin and watches a fronto-parallel screen at a distance of 150 cm. (B) Top view of Fig. 5 A.

occurs, then the vertical headcentric disparity is non-zero. A corrective term, which is related to vertical disparity and depends on the type of error, is added to the horizontal headcentric disparity in order to correct for the oculomotor error. Thus, the original headcentric model relies entirely on the retinal signals. However, a non-zero vertical headcentric disparity field may also be caused by a retinal error. Thus, the visual system may not rely entirely on the retinal signals. It is feasible that the visual system weights the oculomotor signals and the retinal signals according to their reliability. This weighting can be realised by multiplying the different correction terms by weighting factors given by the ratios that we measured. Each type of vertical disparity field has its own correction term and its own weighting factor. The fact that the three weighting factors differ from each other within individual subjects suggests that the three vertical disparity fields are processed separately. Each vertical disparity field, signalling a certain type of error in the oculomotor signal, has its own weighting factor; this indicates that the visual system assigns different levels of reliability to different oculomotor signals.

Summarising, we suggest that weighting of disparity and eye position signals occurs. Weighting depends on the type of vertical disparity. The present results do not tell us whether disparity is of retinal or headcentric nature. Furthermore, these results do not tell us whether weighting occurs at the level of depth maps or at the level of disparity and oculomotor signals.

#### 4.4. Deviation from the cylinder percept

We found that two subjects did not perceive the

quadratic mix transformation as a vertical cylinder, but they perceived it as a sort of hill or valley. The hill or valley was perceived as more curved in the horizontal direction than in the vertical direction. Adams et al. (1996) mentioned the same effect. They did not offer an explanation. We will show here that the effect is explained by the modification of the headcentric model suggested in the previous section. The subjects in our study, who reported the effect (HW and LW), had low ratios in Expt MIX (Table 1). An interpretation in terms of the modified headcentric model is that the vertical disparity is given a low weighting relative to the oculomotor signals. To show the validity of this interpretation we performed the following simulation with the help of the headcentric model.

Percepts, as predicted by the model of Erkelens and Van Ee (1998), were computed for two vertical quadratic mix transformations (factor 0.04 and  $-0.04$ ). The correction term based on vertical disparity indicates how large the correction for oculomotor errors should be. The subject would perceive a fronto-parallel plane if the visual system used this correction in a proper way. By multiplying the correction term by a small weighting factor (0.3 in this simulation) the vertical disparity becomes underweighted. Fig. 5 shows the reconstructed surfaces. The surfaces are curved in two directions. They are more curved in the horizontal direction than in the vertical direction, just as the subjects reported. The curvature of the surfaces in the horizontal direction is less in the middle of the surface than at the upper and the lower part of the surface.

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## Appendix A

### A.1. Expression of vertical quadratic mix transformation

We used a vertical transformation of the left and right eye's image on a screen, which induces an elevational headcentric disparity field identical to the field evoked by a horizontal vergence signal error. We will refer to this transformation as quadratic mix. Rogers and Bradshaw (1995) and Adams et al. (1996) also used this transformation. They designed a transformation, which scales the vertical retinal disparity. Such stimulus depends on the fixation point. Only when the fixation point is straight ahead, the scaling of vertical retinal disparity agrees with the vertical quadratic mix trans-

formation on the screen. To avoid confusion we refer to our transformation as quadratic mix.

In order to obtain the expression for quadratic mix, we considered a vertical scale with a magnification of  $b$  first Fig. 6:

$$\begin{cases} y_L = y + \frac{1}{2}by \\ y_R = y - \frac{1}{2}by \end{cases} \quad (1)$$

If the co-ordinate systems are chosen as described by Erkelens and Van Ee (1998), this transformation results in a vertical disparity field identical to the field evoked by a horizontal version signal error (with a gradient in the vertical direction).

Secondly, a vertical shear over an angle  $\beta$  ( $\tan(\beta) = c$ ) was considered (Fig. 6):

$$\begin{cases} y_L = y + \frac{1}{2}cx \\ y_R = y - \frac{1}{2}cx \end{cases} \quad (2)$$

This transformation on the screen induces a vertical disparity field identical to the field evoked by a cy-

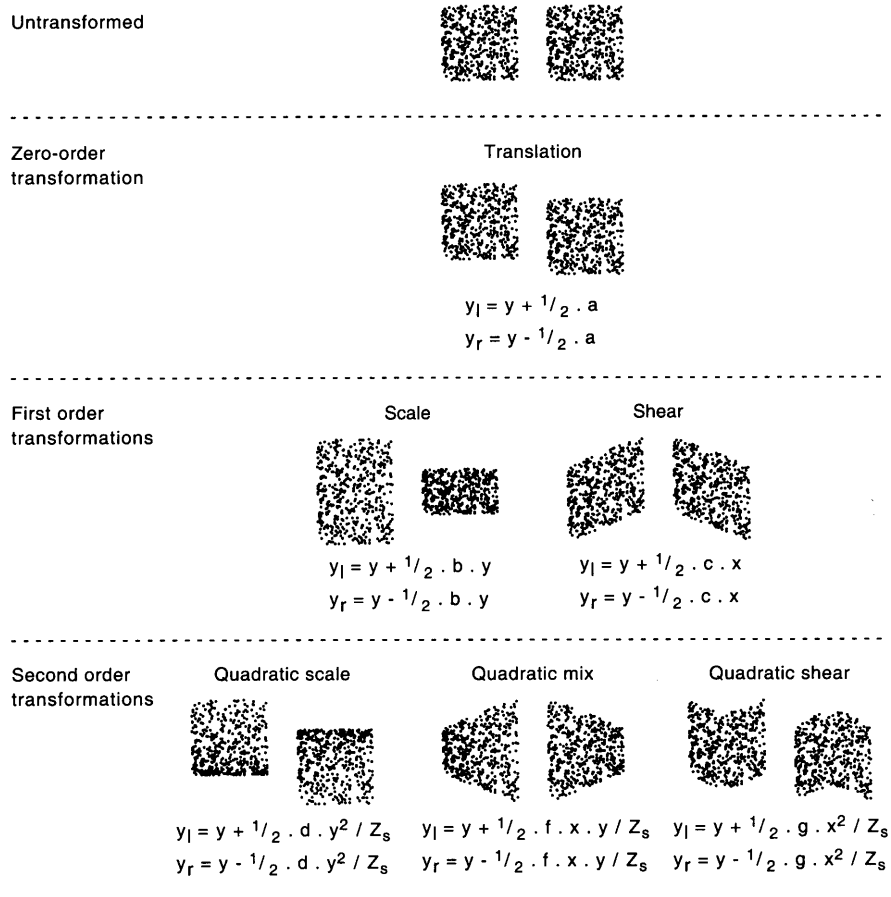


Fig. 6. Different types of vertical transformations of stereograms on a fronto-parallel screen. The transformations can also be performed in the horizontal direction or in both the horizontal and the vertical direction. Zero and first-order transformations of the two half-images are commonly used in studies of stereoscopic depth perception. The second and higher order transformations are rarely used.  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $f$  and  $g$  are constants.  $x$  and  $y$  are the points in the untransformed stereogram.  $x_l = x_r = x$  in all vertical transformations.

clovergence signal error (with a gradient in the horizontal direction).

The expression found for the vertical quadratic mix transformation is straightforward. This transformation can be considered as a vertical scale (a transformation depending on the  $y$ -co-ordinate) which depends on the horizontal ( $x$ ) co-ordinate. Thus, the expression is obtained by multiplying  $x$  by  $y$ :

$$\begin{cases} y_L = y + \frac{1}{2}fxy/z_s \\ y_R = y - \frac{1}{2}fxy/z_s \end{cases} \quad (3)$$

Dividing  $fxy$  by the screen distance ( $z_s$ ) renders the constant  $f$  dimensionless.

This transformation is a second-order transformation. There are three second-order transformations possible (Fig. 6). Firstly, a quadratic scale transformation is a non-linear scale, which depends on the vertical distance ( $y$ ). Secondly, a quadratic shear transformation is a non-linear shear, which depends on the horizontal distance ( $x$ ). Thirdly, quadratic mix is a transformation that lies in between quadratic scale and quadratic shear.

#### A.2. Expression of a quadratic horizontal scale

The expression for horizontal quadratic scale is analogous to the expression for vertical quadratic scale in which  $y$  is replaced by  $x$ :

$$\begin{cases} x_L = x + \frac{1}{2}hx^2/z_s \\ x_R = x - \frac{1}{2}hx^2/z_s \end{cases} \quad (4)$$

A horizontal quadratic scale induces an ellipse or a hyperbola, depending on the sign of the quadratic scale factor,  $h$ .

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