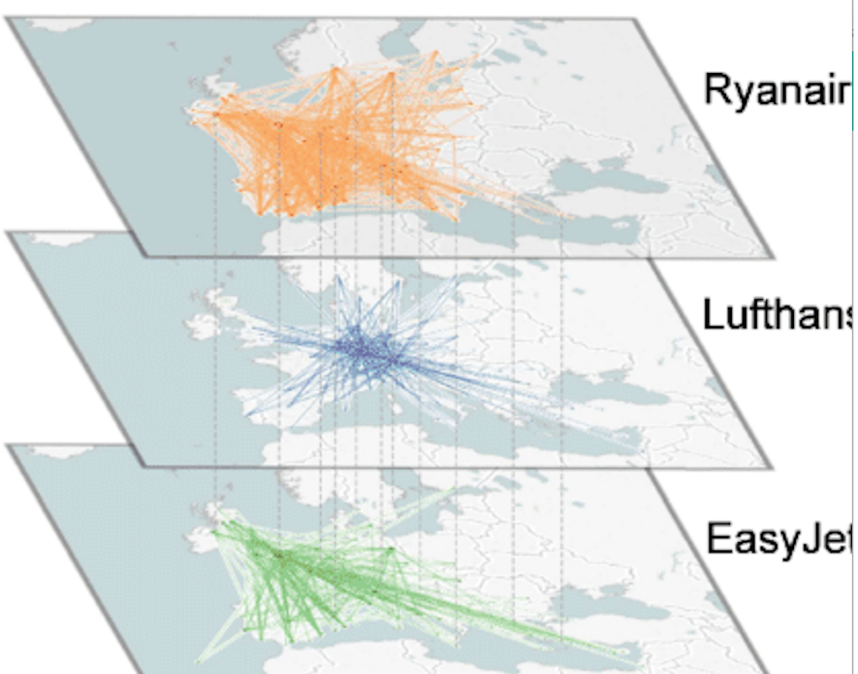
Multilayer Network Visualization

Course : Data Visualization
Lecturer : Tamara Mchedlidze
Utrecht University, Dept. of Information and Computing Sciences



Wu et al, 2017


Telea et al, 2011

Lecture Overview

- Multilayer network
- Visualization types for multilayer networks
- Algorithm for visualization in 2.5D
- Edge simplification - bundling
- An algorithm for edge bundling
- Proposed technique for the implementation

Adding complexity

- definition of network/graph we used till now (nodes, edges and perhaps labels) is a simplification of reality, where the network are often way more complex

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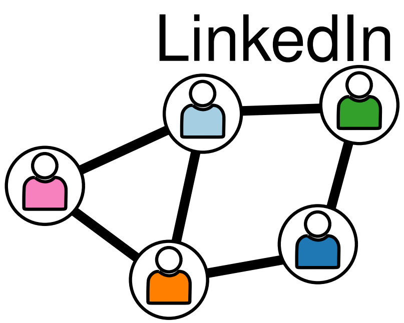

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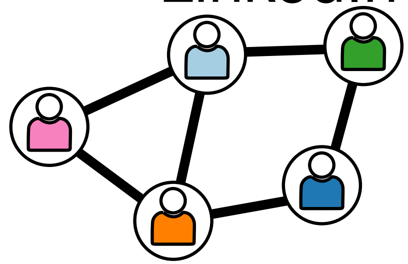
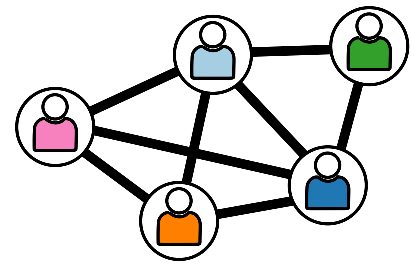
LinkedIn





connections
in a company

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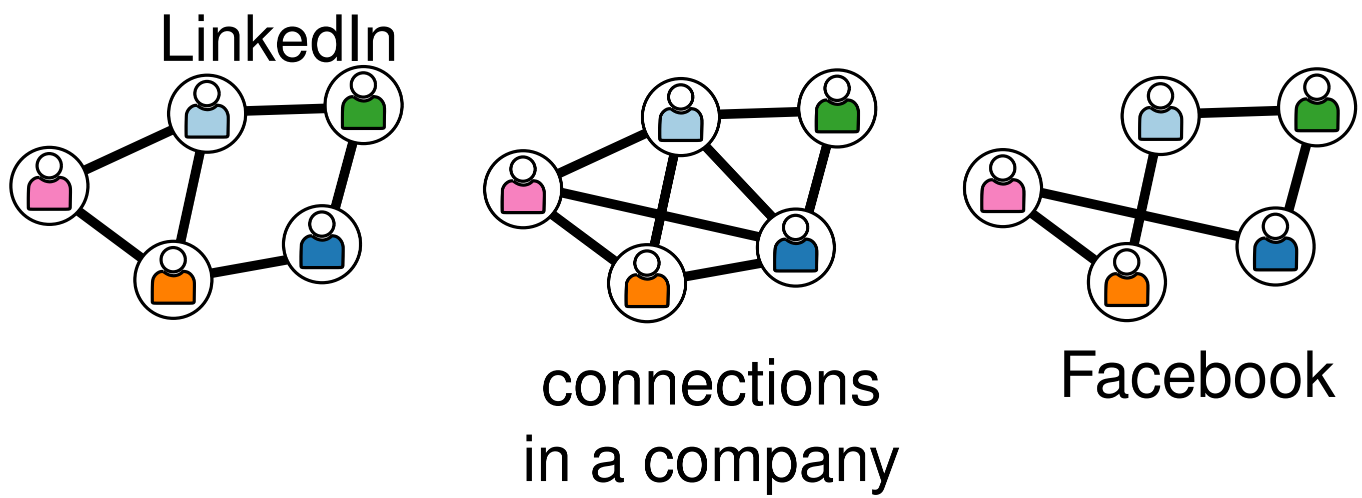

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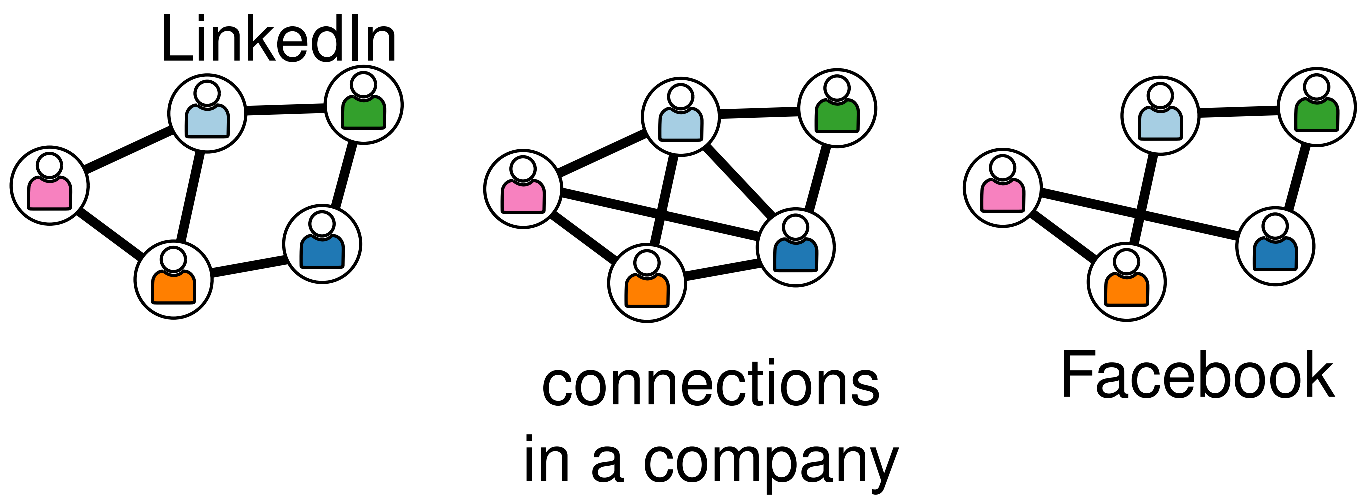

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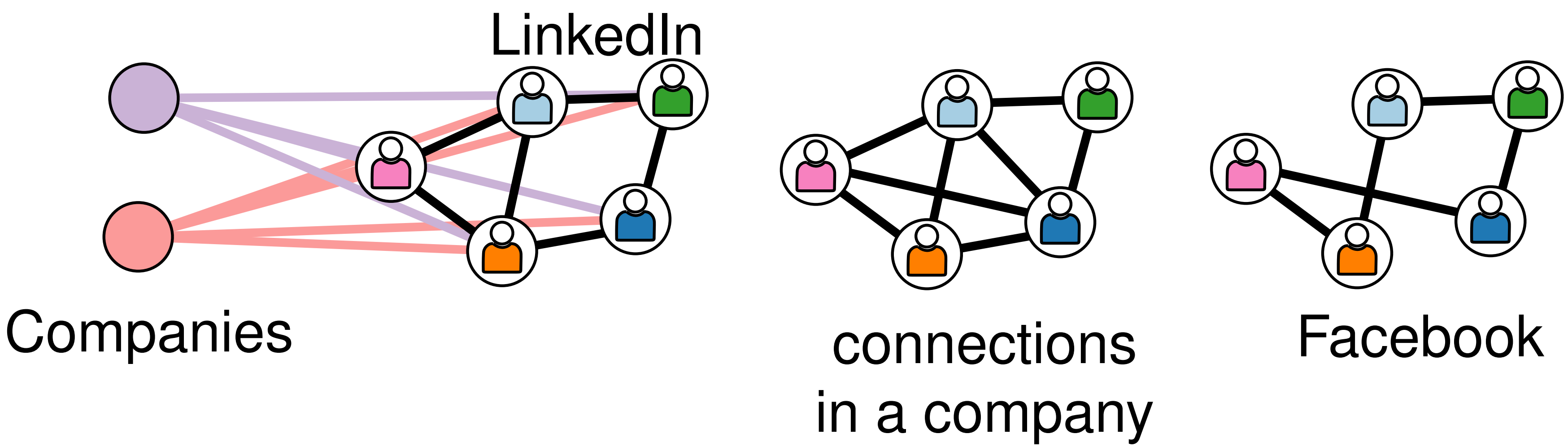

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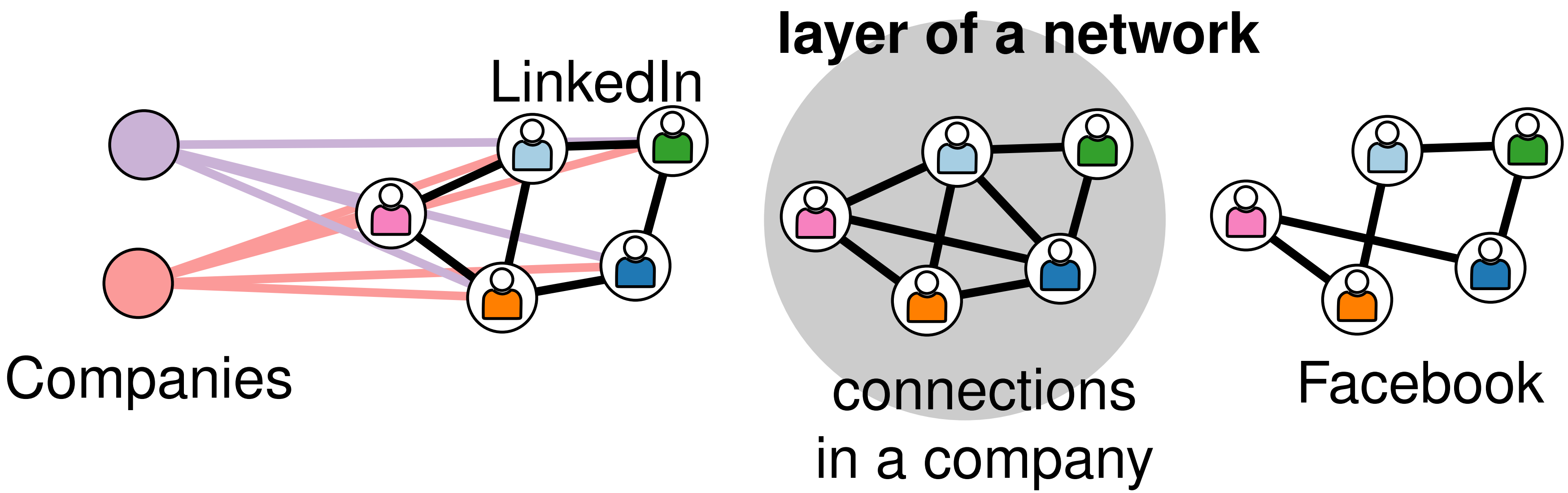

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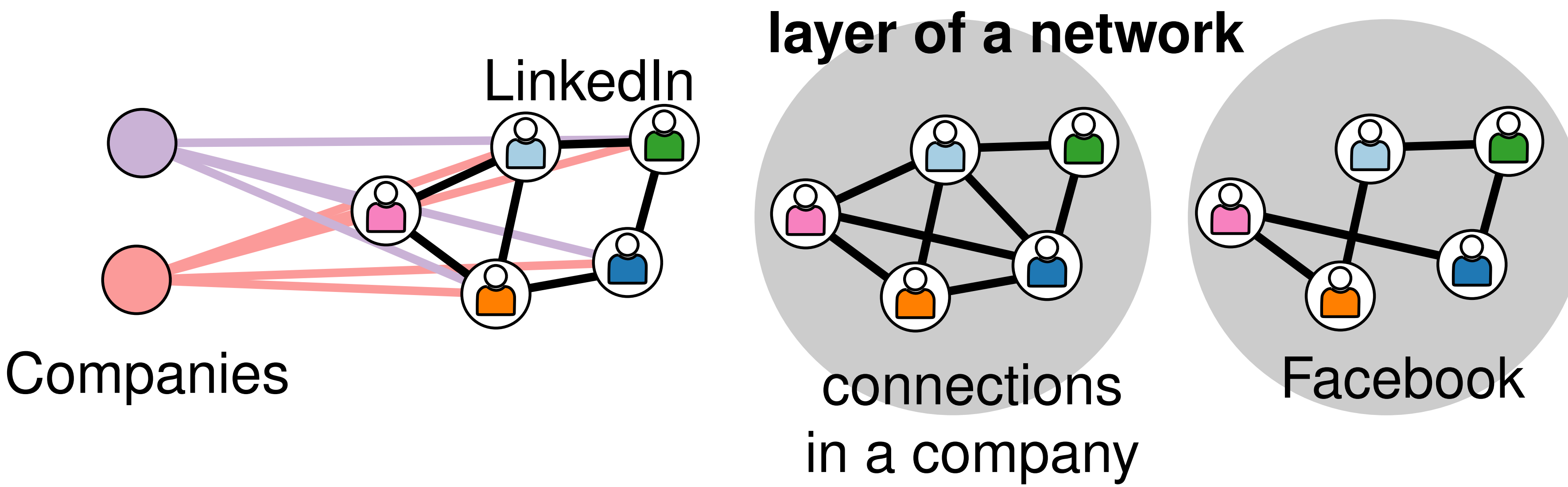

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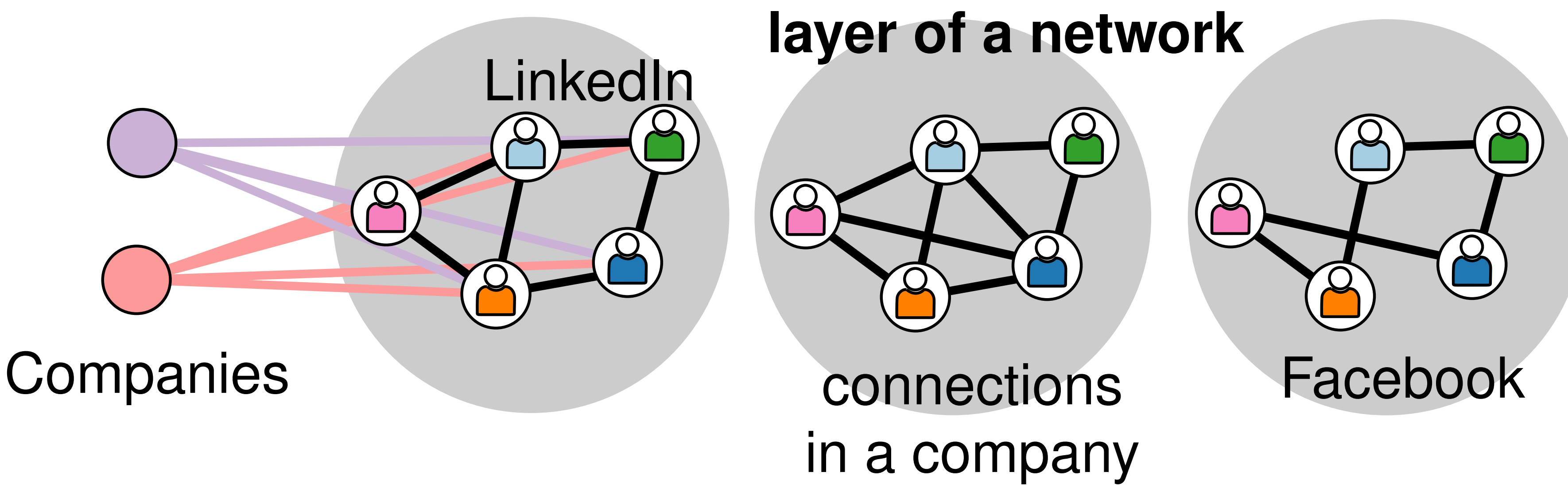

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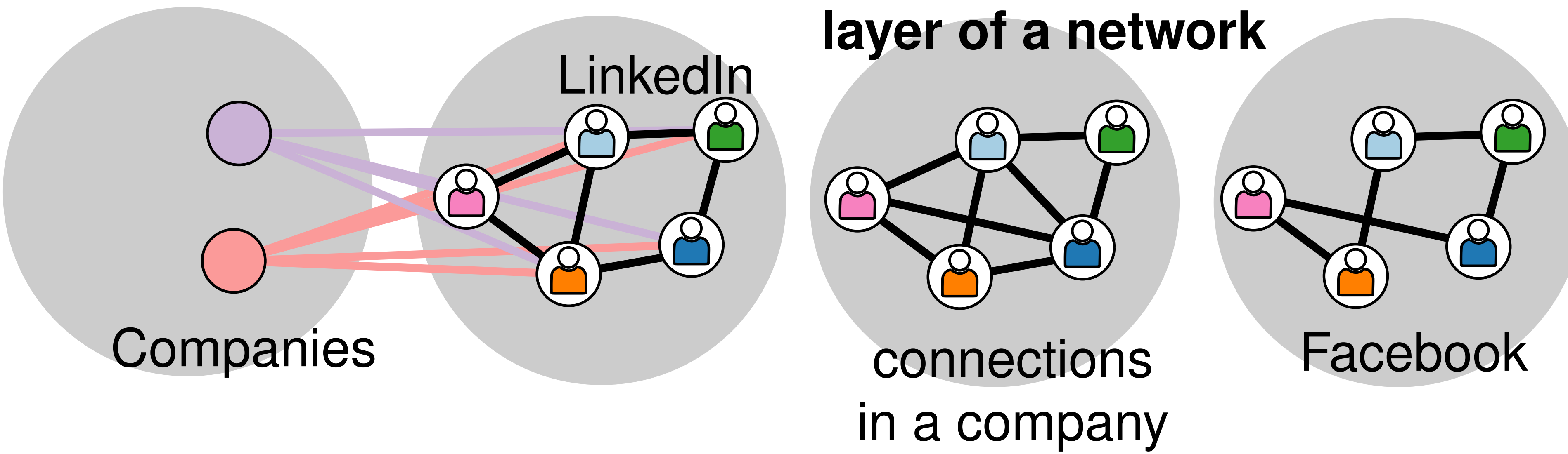

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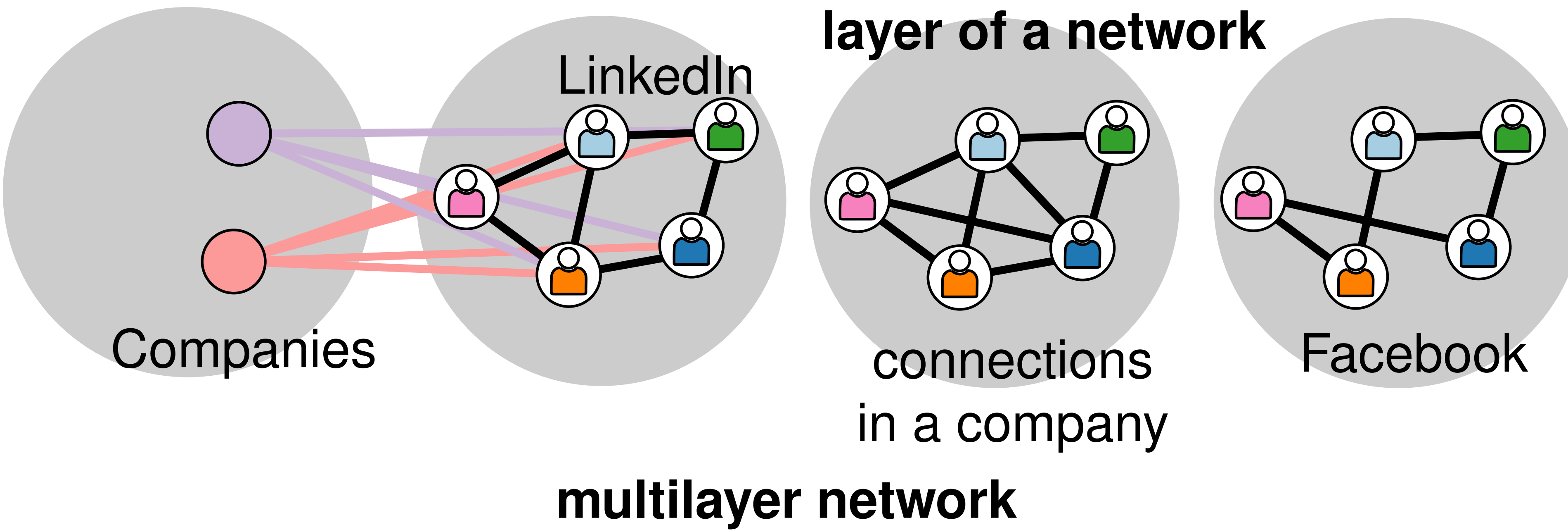

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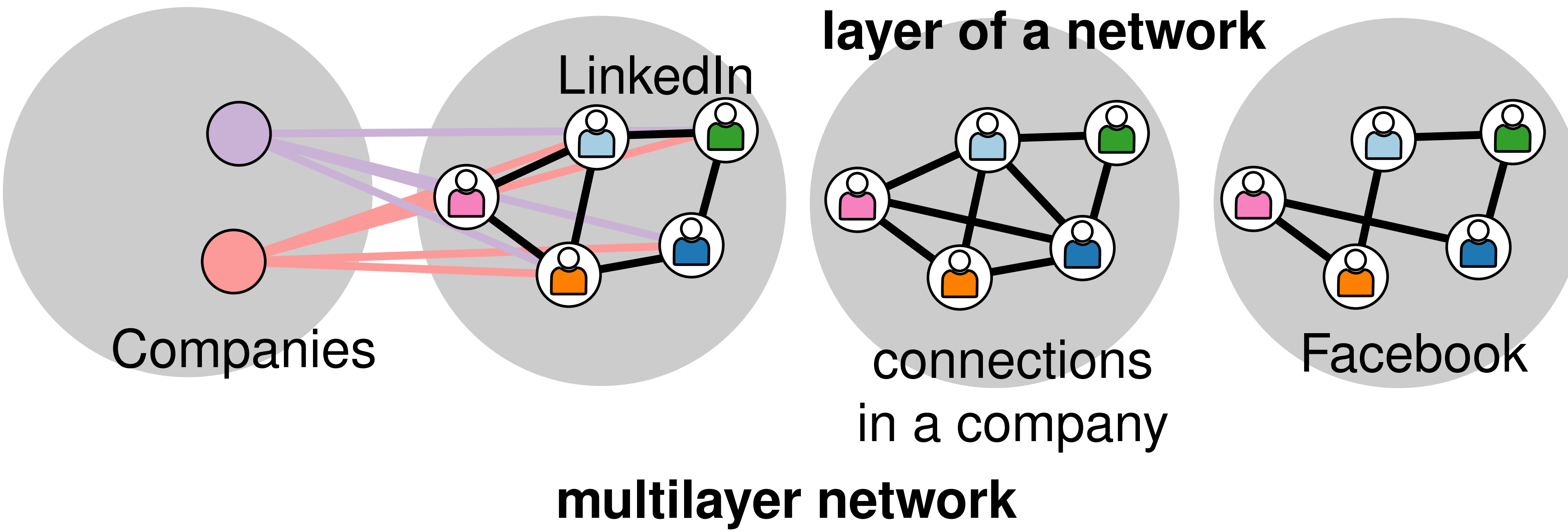

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- nodes of other types
- analysis of graph patters across layers reveal complex facts about the data

Multilayer Network

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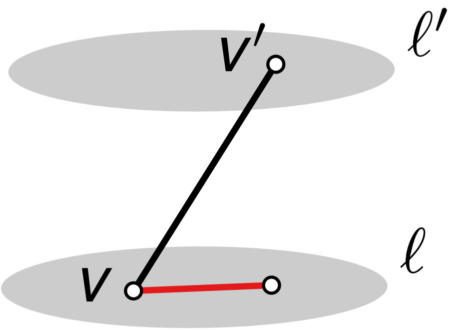

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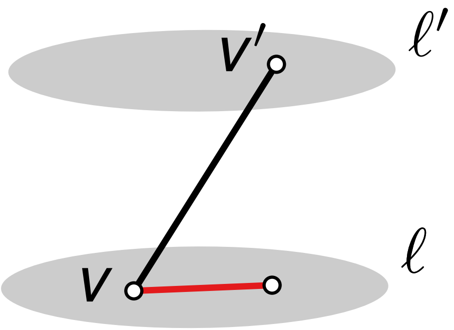

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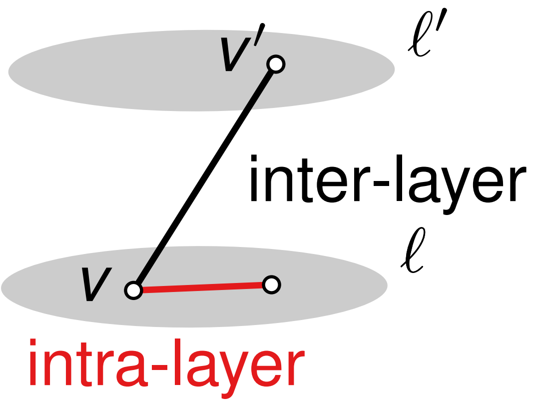

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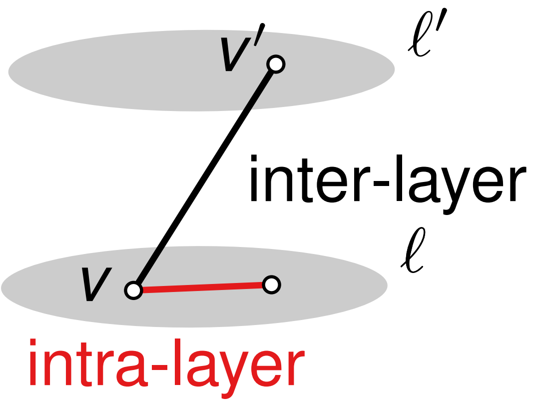

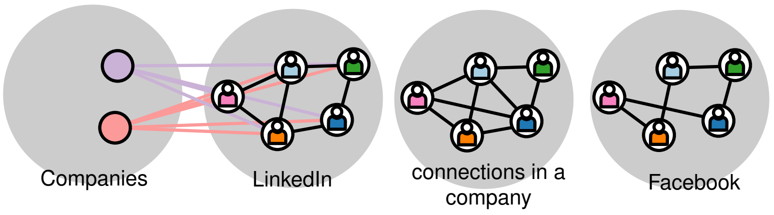

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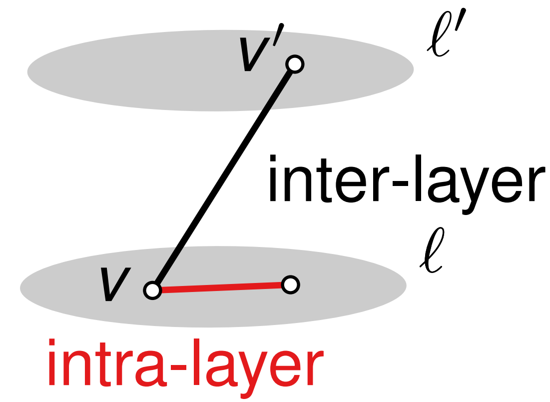

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  ℓ_1 =LinkedIn,
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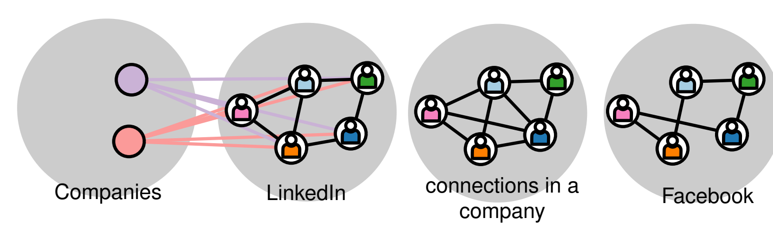

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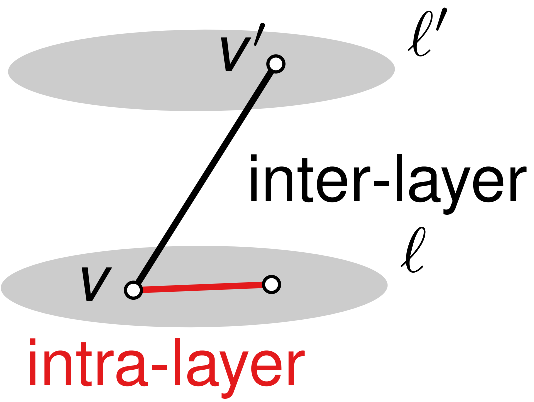

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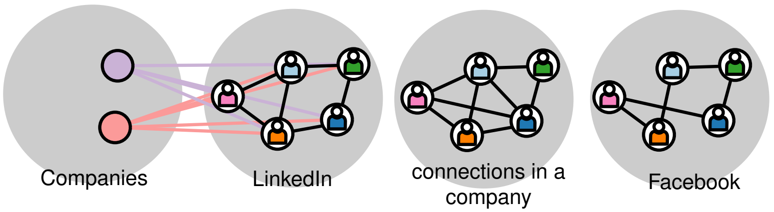

- node v = appears as (v, ℓ_1), (v, ℓ_2), (v, ℓ_3) in V_m

Multilayer Network

multilayer networks appear as models in

- biology: genomic, proteomic and metabolomic data to model intricate biological processes

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- biology: genomic, proteomic and metabolomic data to model intricate biological processes
- civil infrastructure: urban planning taking into account the interplay between multiple networks such as transportation networks, energy networks, telecommunication networks and water/wastewater networks
- epidemiology, sociology (including criminology), digital humanities

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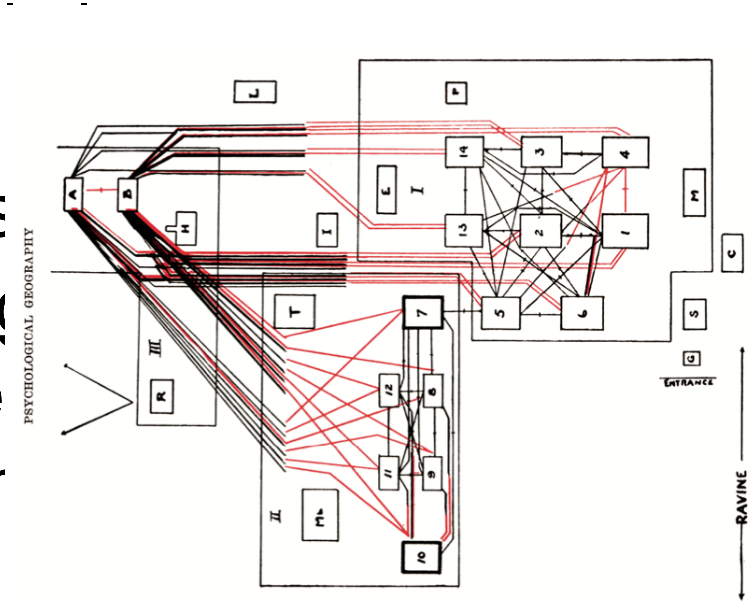

date back to 50s – notion of many relationships between individuals in the sociograms introduced by Moreno

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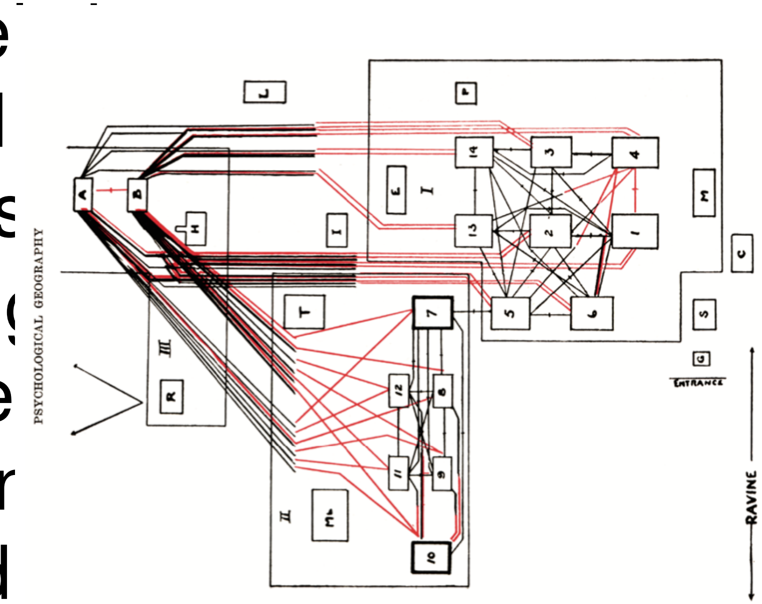

date back to 50s – notion of many relationships between individuals in the sociograms introduced by Moreno

many names: multi-label, multi-edge, multirelational, multiplex, heterogeneous, multimodal, multiple edge set networks, interdependent networks, interconnected networks, networks of networks, ... – unified under a single framework by Kivelä et al. 2014

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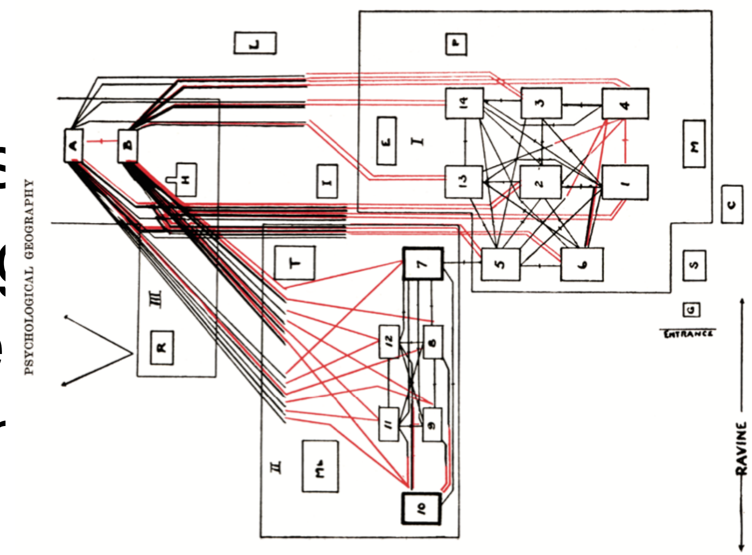
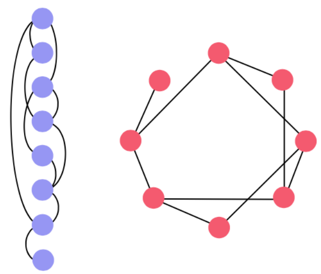


Have you seen relational data that need to be modeled as multilevel networks?

networks, networks of networks, ... – unified under a single framework by Kivelä et al. 2014

Multilayer Network Visualizations

Types of visualizations of multilayer networks



1-dimensional: circular, linear

- 1-dimensional representations rely on Gestalt principle of continuation to perceptually group the layers

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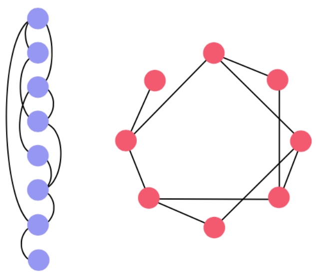

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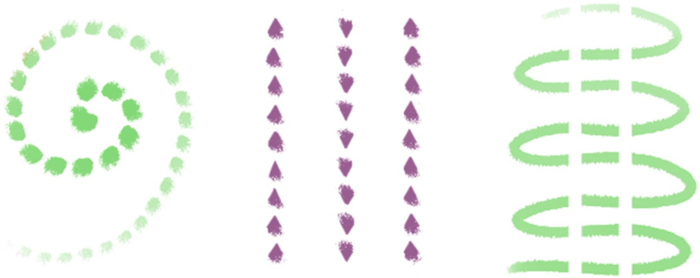

Gestalt principle: continuation, continuity

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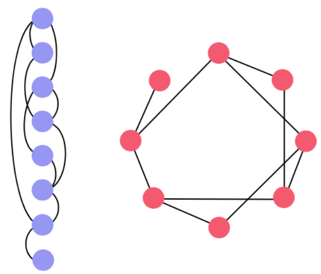

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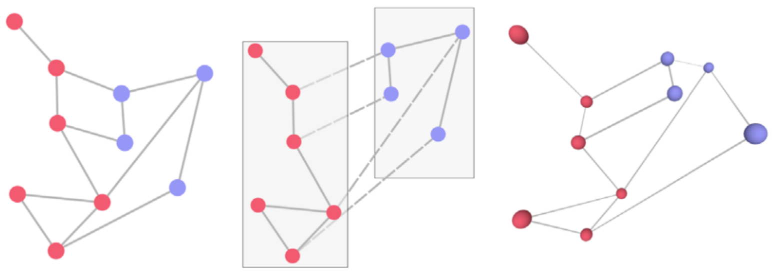

2D, 2.5D, 3D representations

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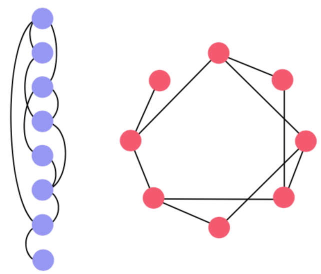

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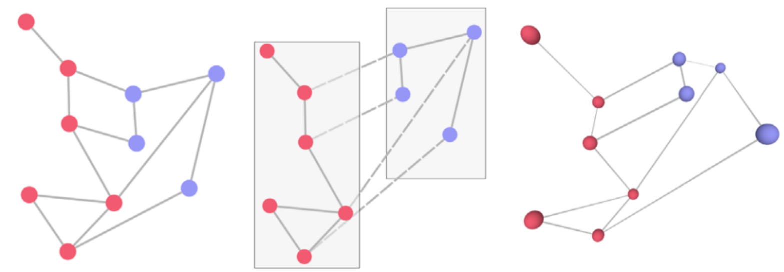

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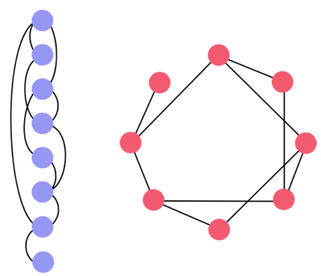

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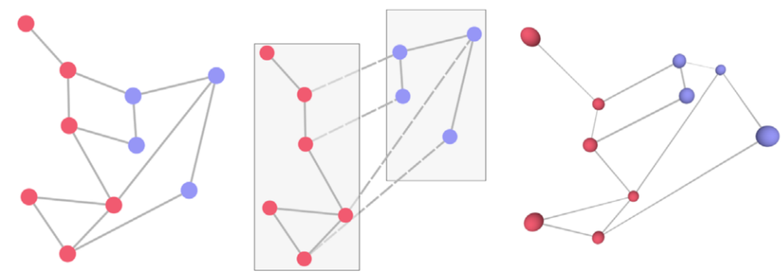

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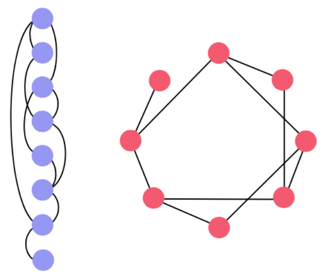

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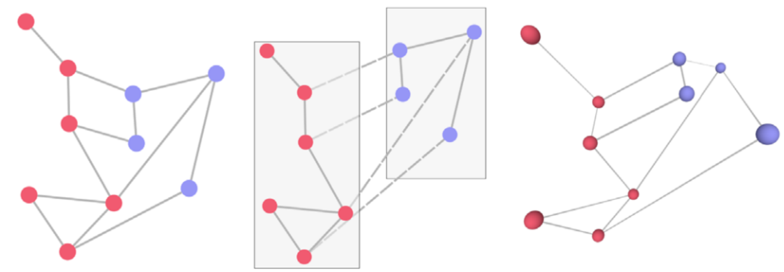

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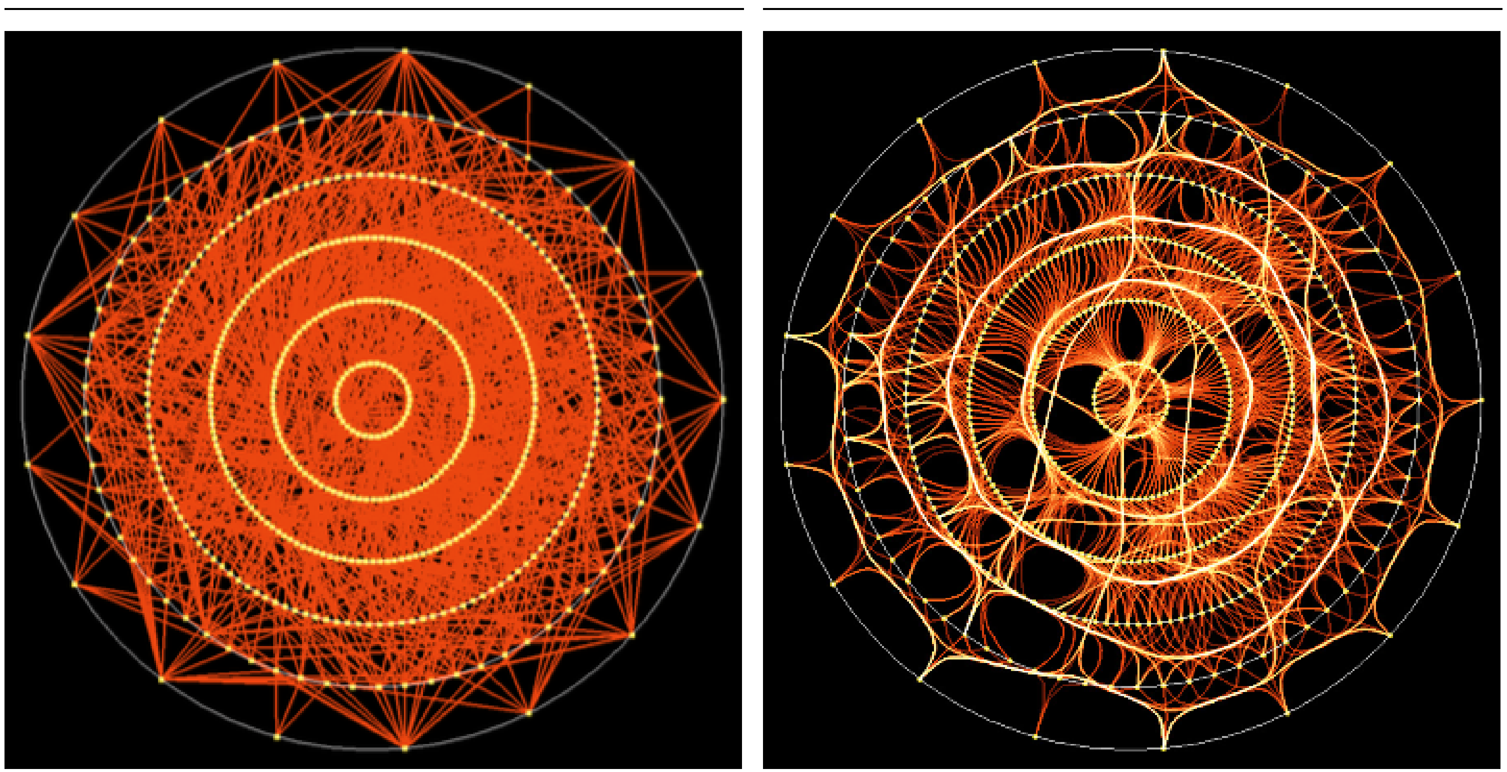
- 3D – depth is indicating the layer, camera movement is necessary

1-dimensional representaiton: circular

- Mushroom data set from the UC Irvine Machine Learning Repository



Visualization of Frequent Itemsets with Nested Circular Layout and Bundling Algorithm, Bothorel et al 2013

1-dimensional representaiton: circular

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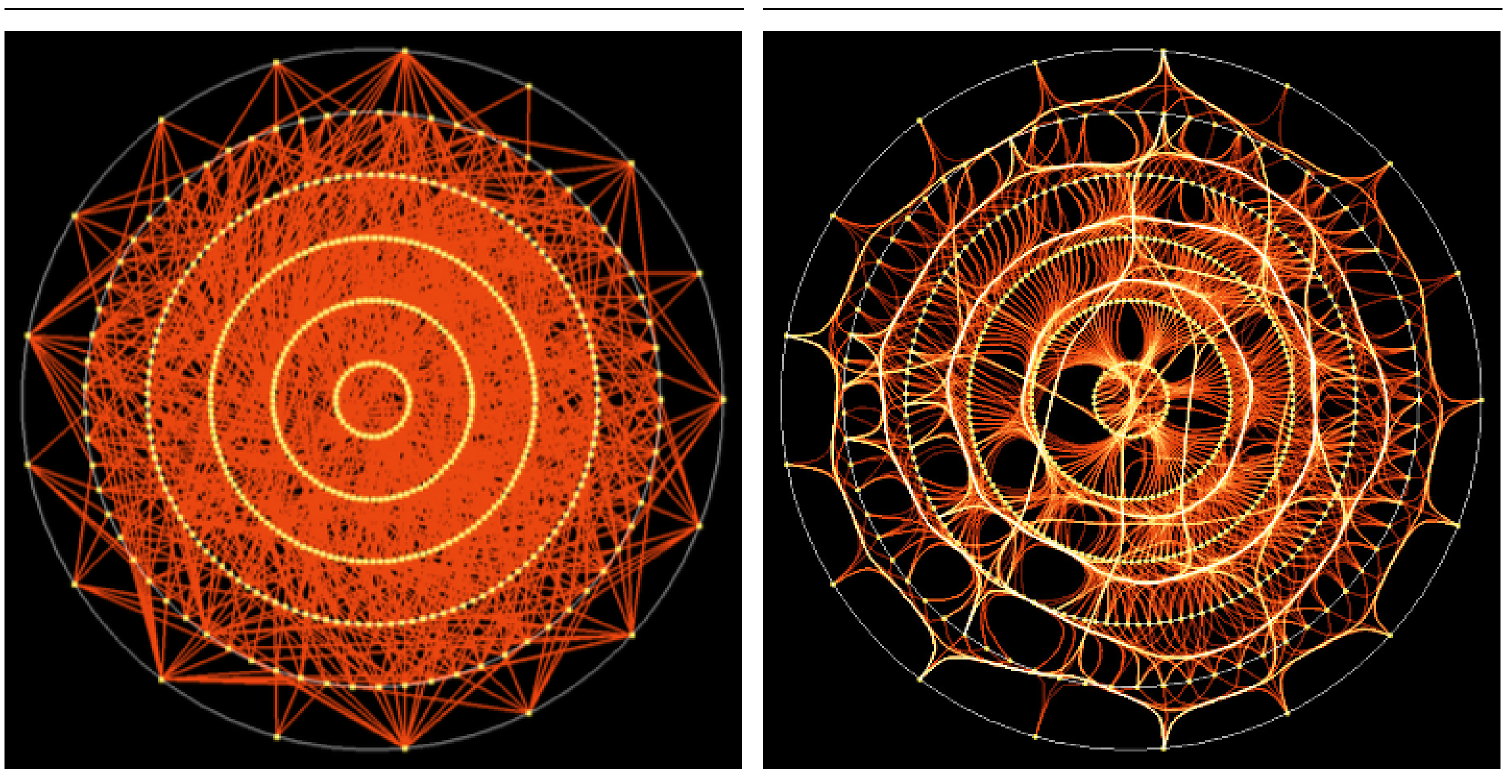

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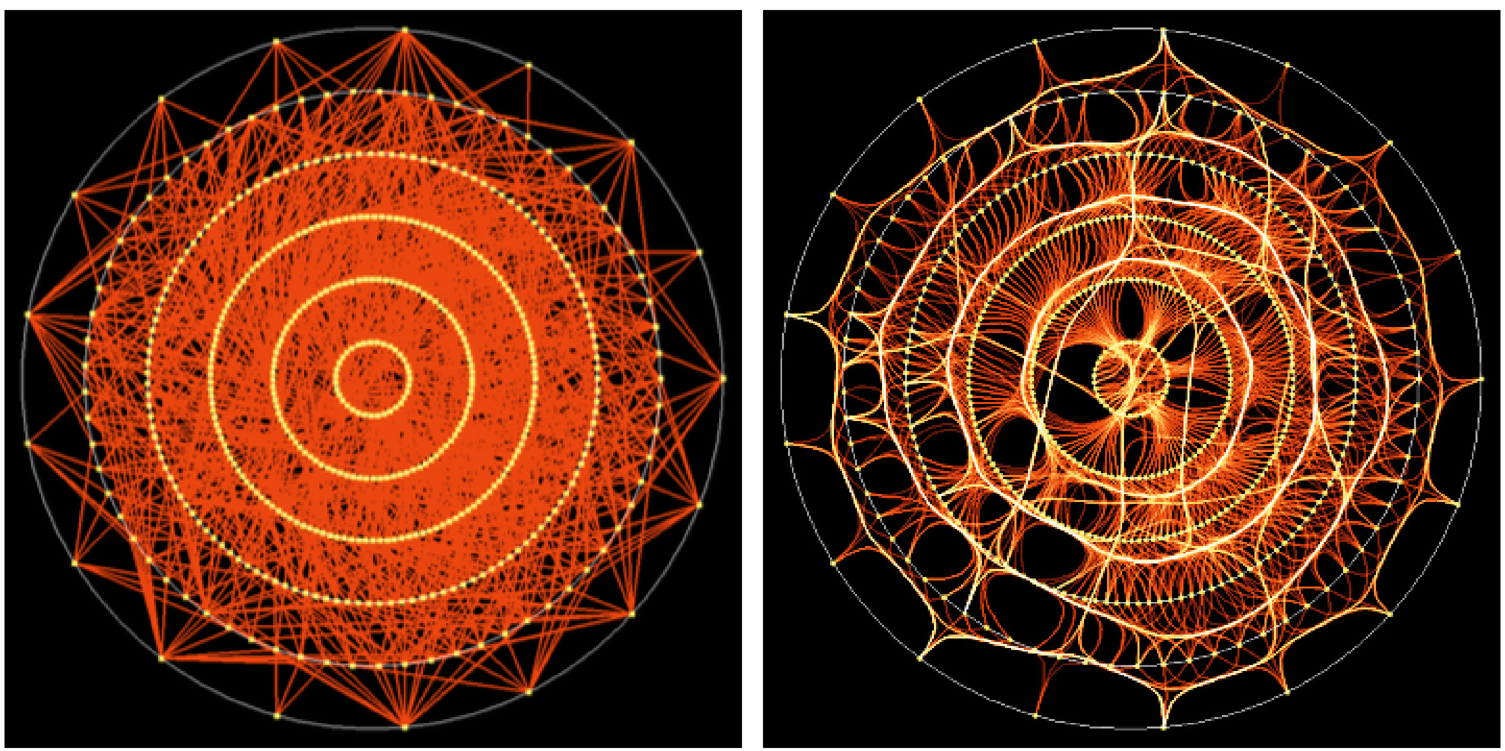
- layers – attributes: the edibility, the cap shape, the odor, the ring, etc.



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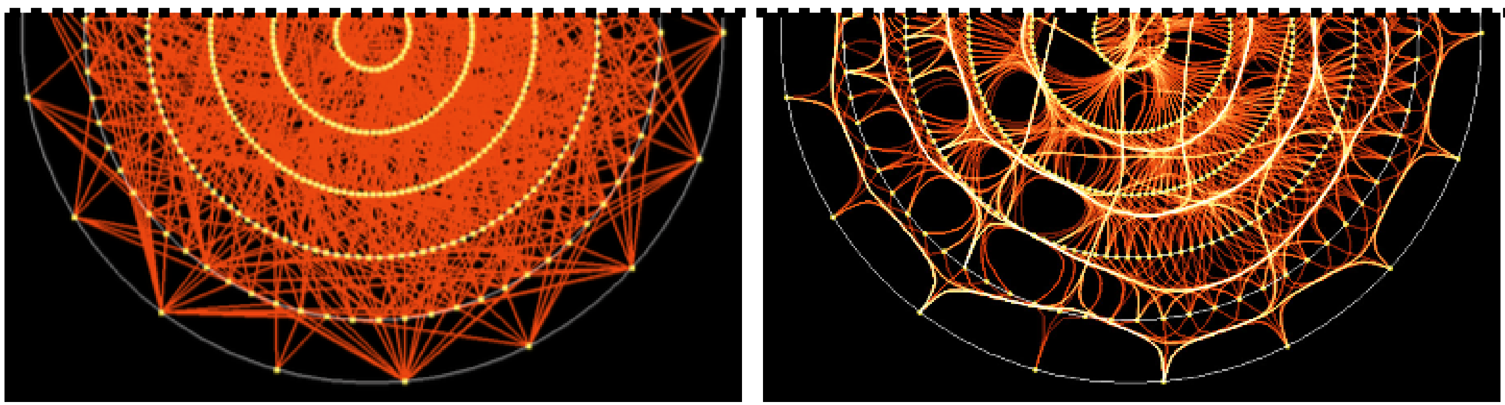
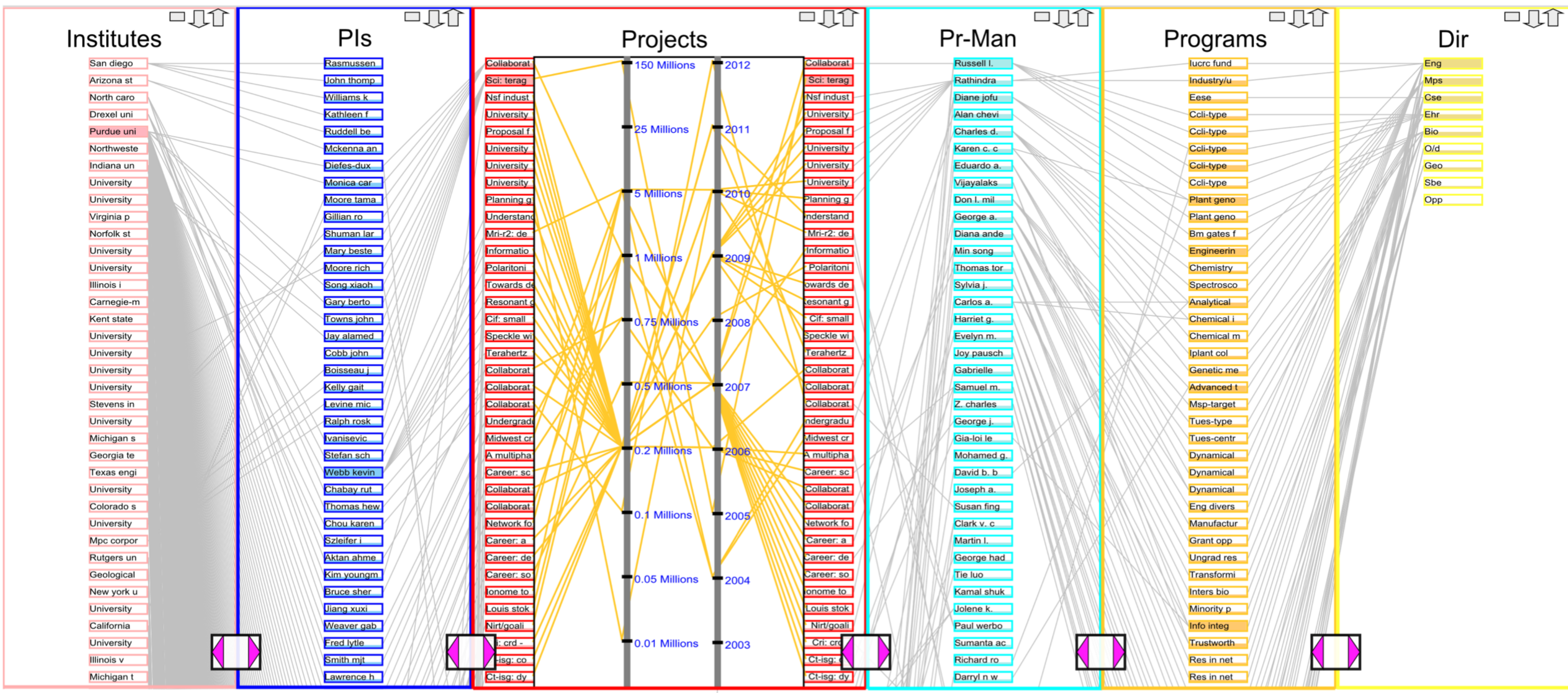
Which of the techniques you know can you use to construct this layout?



Visualization of Frequent Itemsets with Nested Circular Layout and Bundling Algorithm, Bothorel et al 2013

1-dimensional representaiton: linear

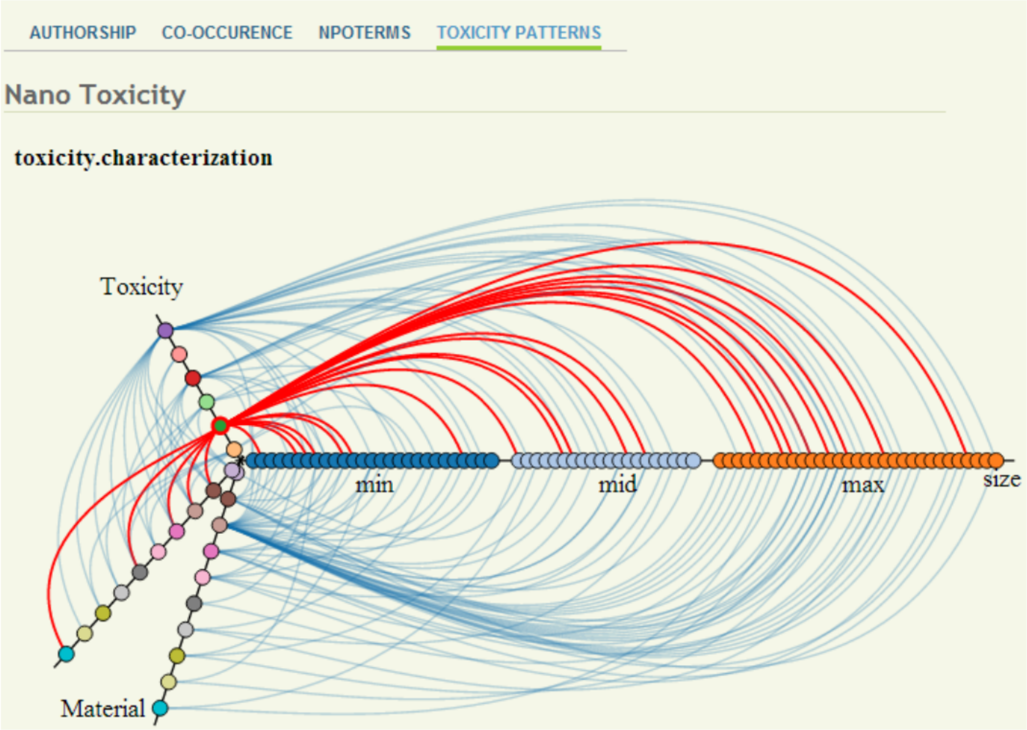
- multimodal NSF funding data consisting of Institutions, PIs (and Co-PIs), Projects, program managers (Pr-Man), NSF programs (Programs), and NSF directorates (Dir)
- remind parallel coordinate plots



Visual Analytics for Multimodal Social Network Analysis: A Design Study with Social Scientists, Ghani et al, 2013

1-dimensional representaiton: linear

- Hive plot: axes are arranged radially
- investigation among nano-toxicity type, nanomaterial and particle size



A user-centred approach to information visualisation in nano-health, Yang et al, 2016

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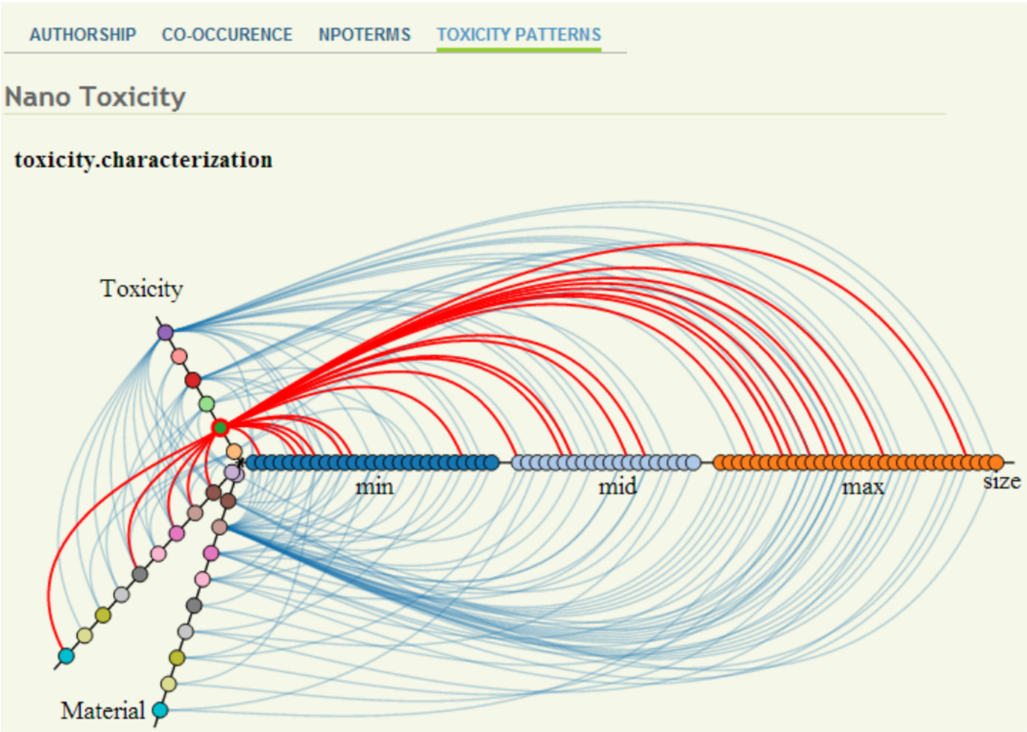

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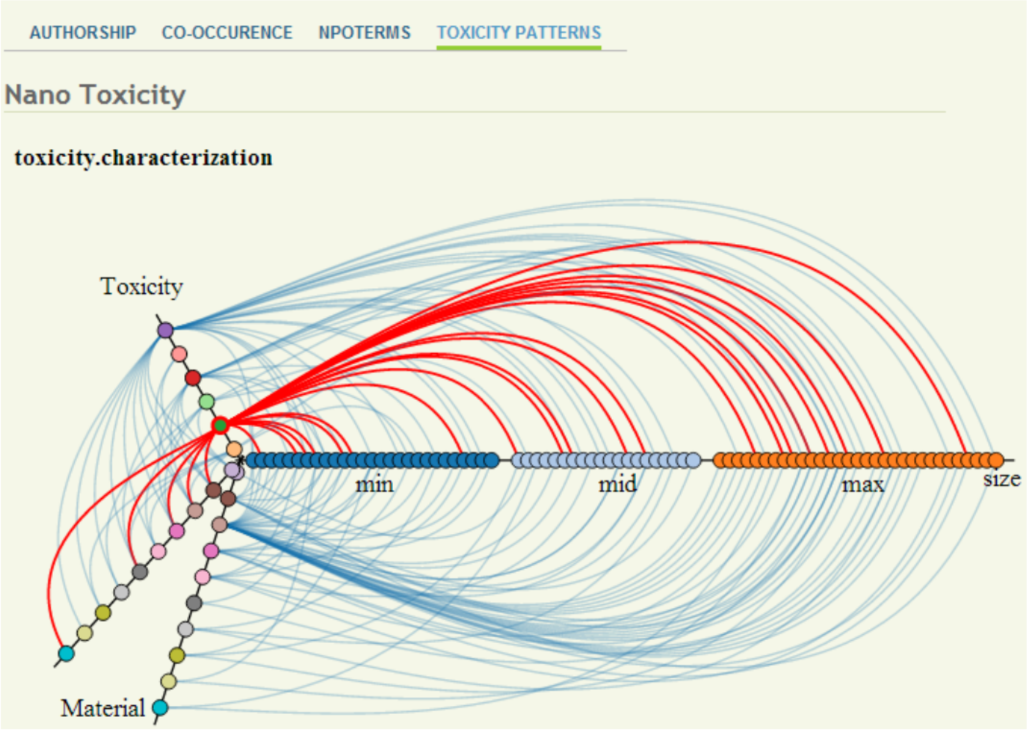
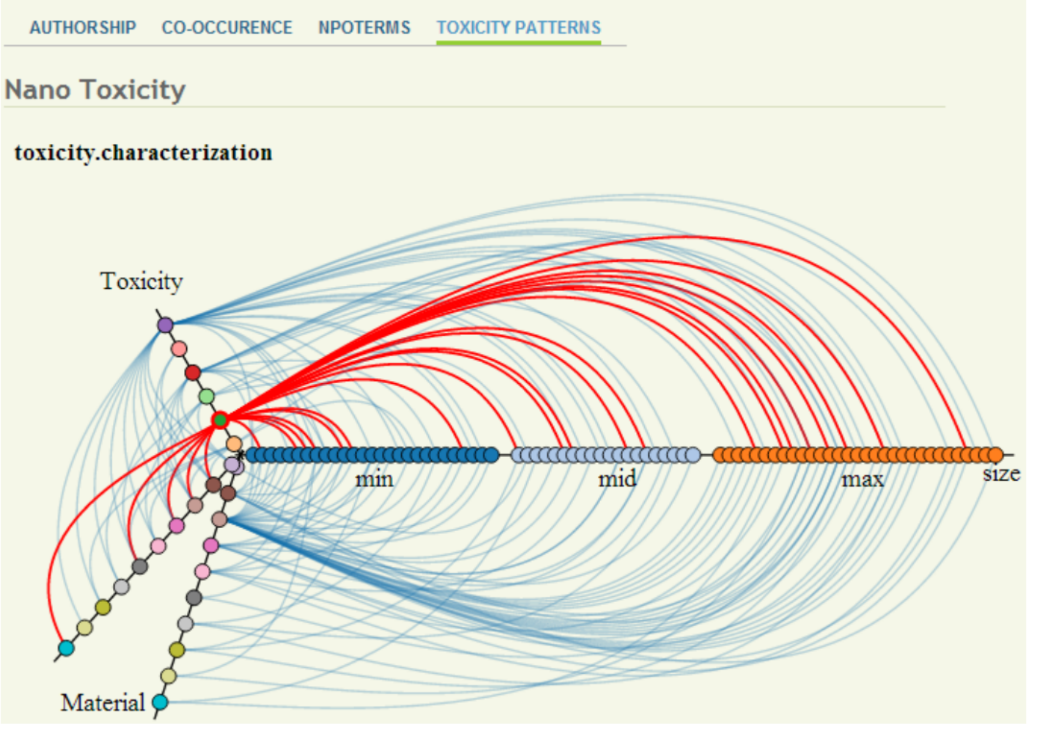
- edges are displayed only between adjacent layers
- nodes are arranged based on graph metric (e.g. degree)



A user-centred approach to information visualisation in nano-health, Yang et al, 2016

1-dimensional representaiton: linear

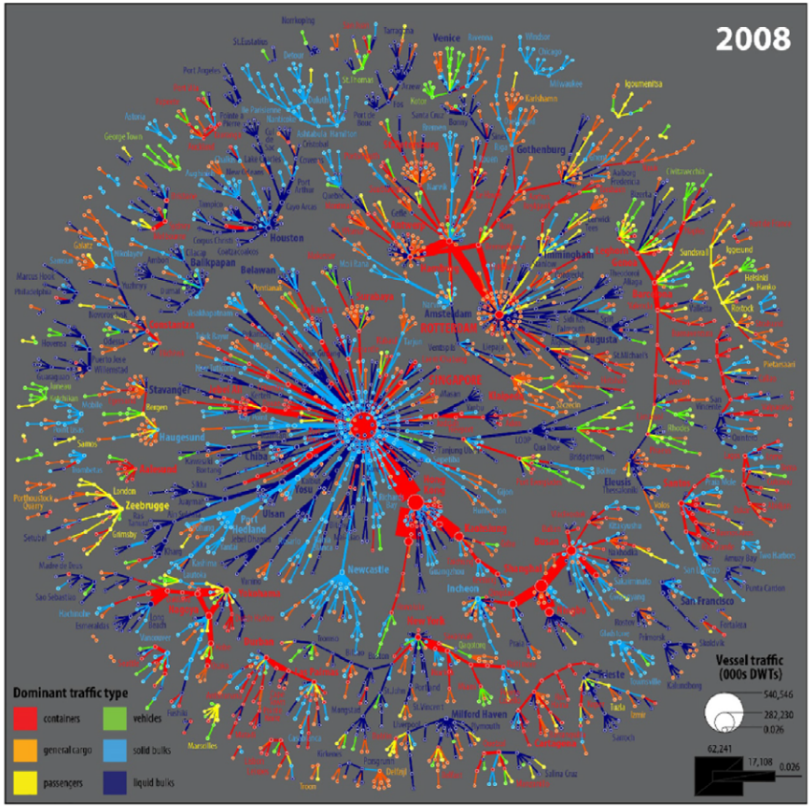
- Hive plot: axes are arranged radially
- investigation among nano-toxicity type, nanomaterial and particle size
- edges are displayed only between adjacent layers
- nodes are arranged based on graph metric (e.g. degree)
- reduce clutter using layer dublication



A user-centred approach to information visualisation in nano-health, Yang et al, 2016

2-dimensional representaiton: color

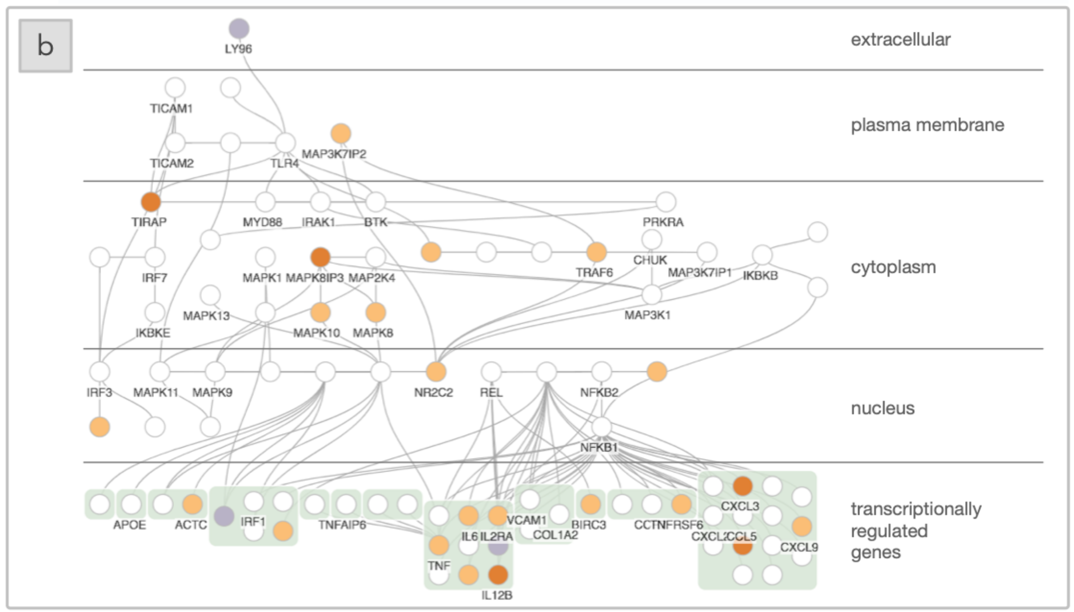
- flow of maritime traffic: nodes represent ports and different edge colours represent different modes of shipping



Multilayer dynamics of complex spatial networks: The case of global maritime flows, Ducuet, 2017

2-dimensional representaiton: separation

- use constrained layouts to separate the nodes of different layers spatially



nodes – physical compounds in a cell; that are separated by physical membranes, creating compartments defining their subcellular location – layeredge; edges interactions among nodes

SetCoLa: High-Level Constraints for Graph Layout, Hoffswell et al, 2018

2-dimensional representaiton: separation

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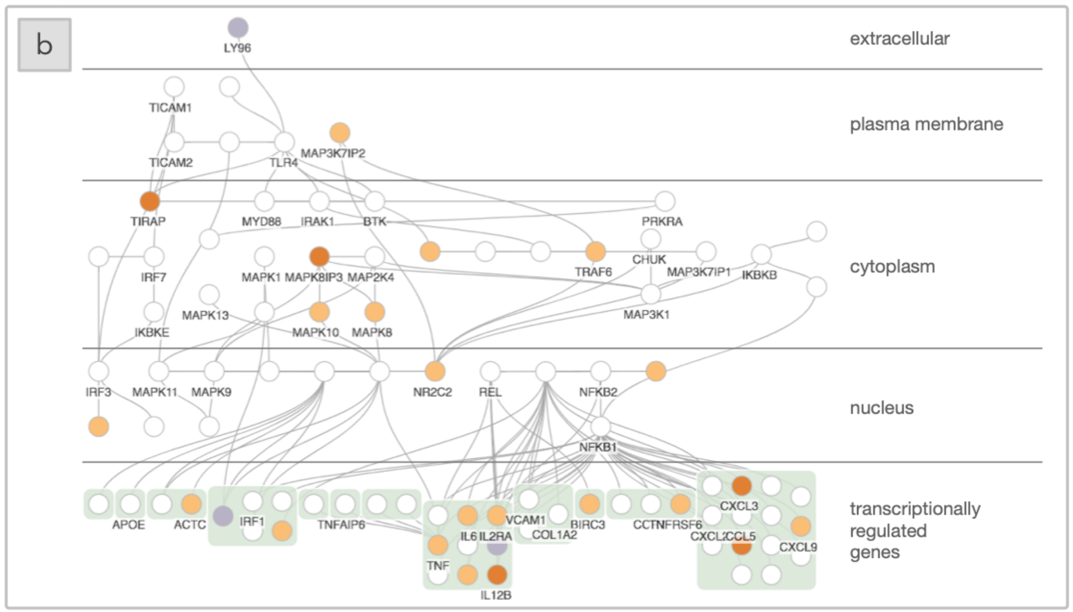

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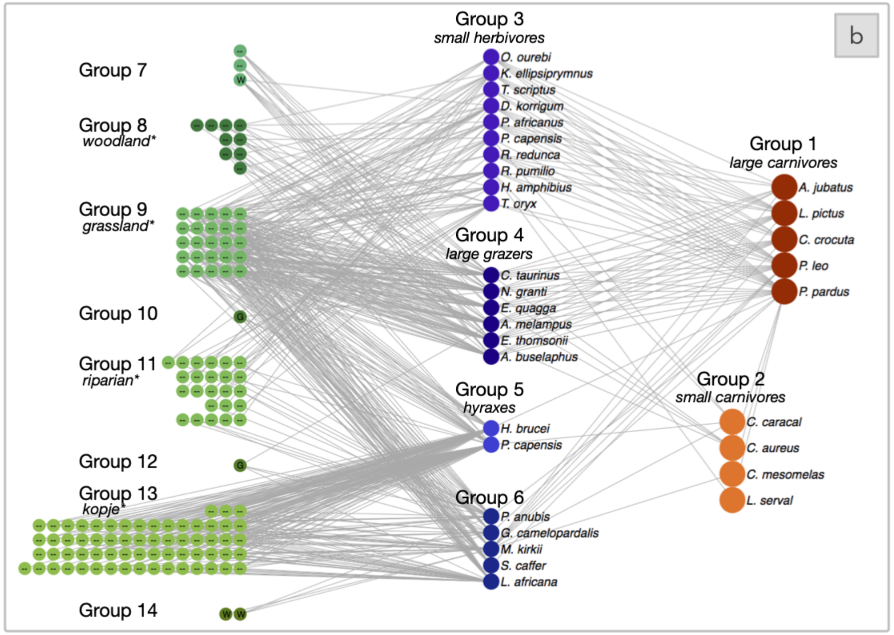

161 plants, herbivores, and carnivores with 592 links between entities – feeding links, groups – clustering, layers – trophic hierarchy

SetCoLa: High-Level Constraints for Graph Layout, Hoffswell et al, 2018

2-dimensional representaiton: separation

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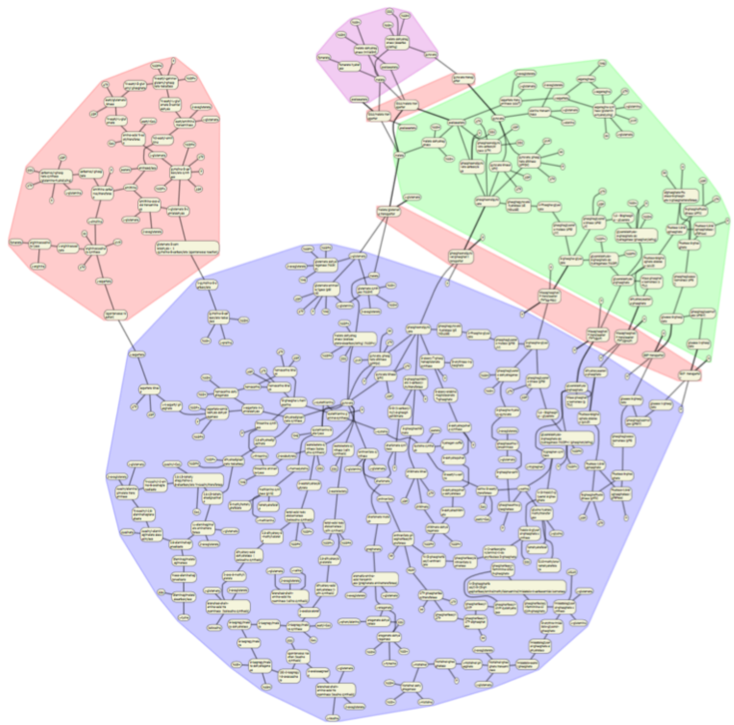
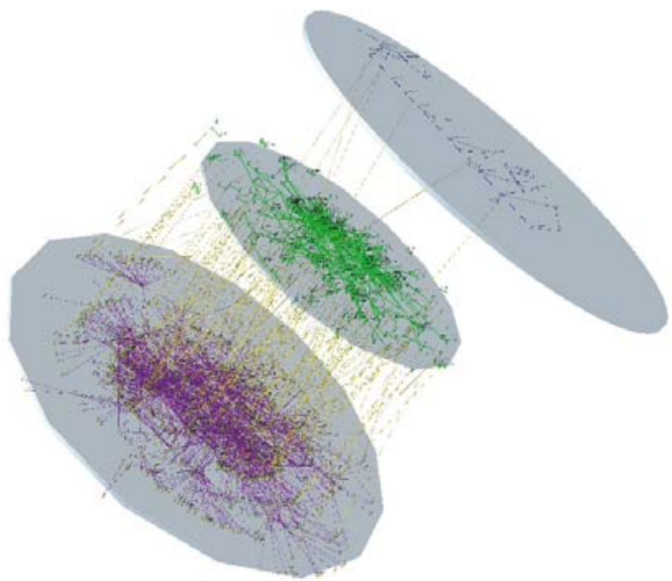


Biological pathways: nodes – proteins, edges–interactions. Rather visualization of clusters, but can be used to show layers too.

Scalable, Versatile and Simple Constrained Graph Layout, Dwyer 2009

2.5-dimensional representaition

- each layer is drawn on a plane and planes are stacked in 3D parallel to each other
- use 2D layout algorithms for a single layer
- same node can appear on many layers – similar positions are desired. Same for reducing edge clutter.

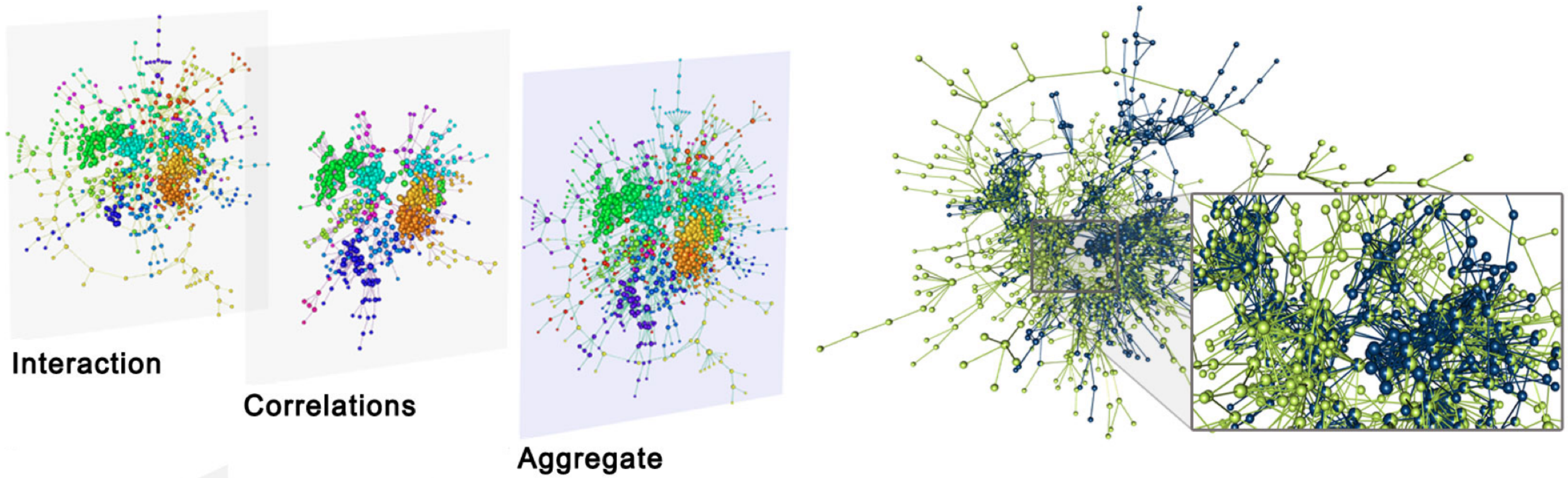


metabolic network, protein interaction networks and gene regulatory network; inter-layer edges: proteins are the result of gene expression, special proteins known as enzymes help transforming metabolites to another.

Visual Analysis of Overlapping Biological Networks, Fung et al, 2009

2.5-dimensional representaition

- same node can appear across layers and lie at the same position
- aggregated layer is possible



- first layer: interaction of genes in Saccharomyces cerevisiae; second layer: genes with similar interaction profiles are connected to each other; third layer: aggregated network
- right - edge colors represent layers

MuxViz: A Tool for Multilayer Analysis and Visualization of Networks, De Do,enico et al, 2015.

2.5-dimensional representaition

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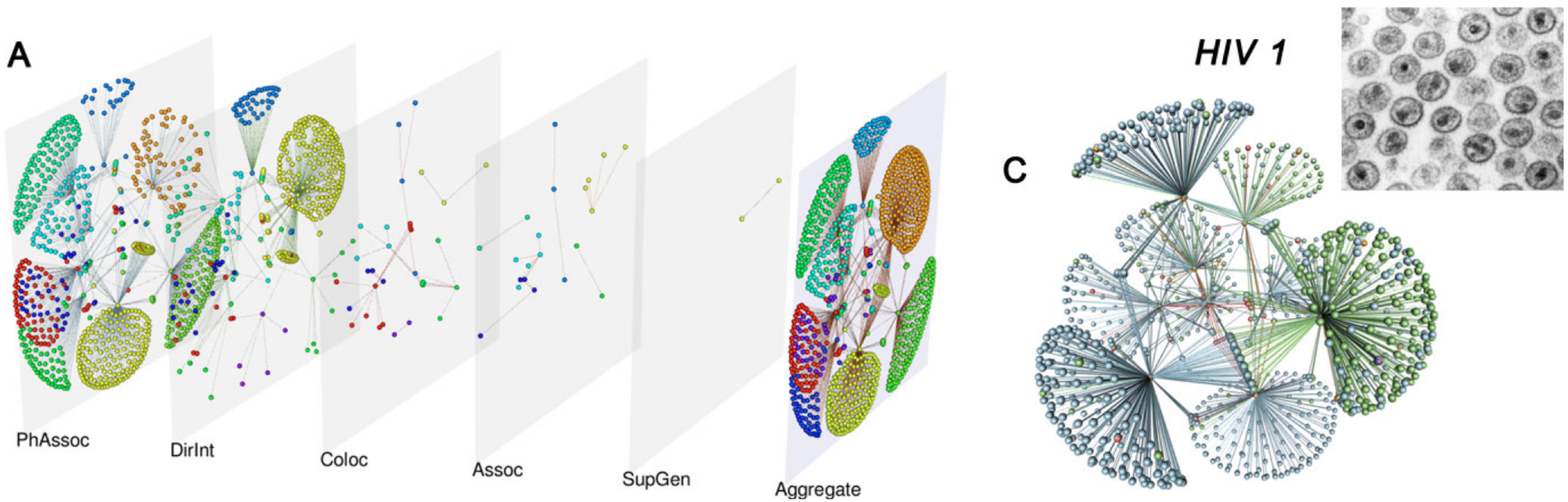
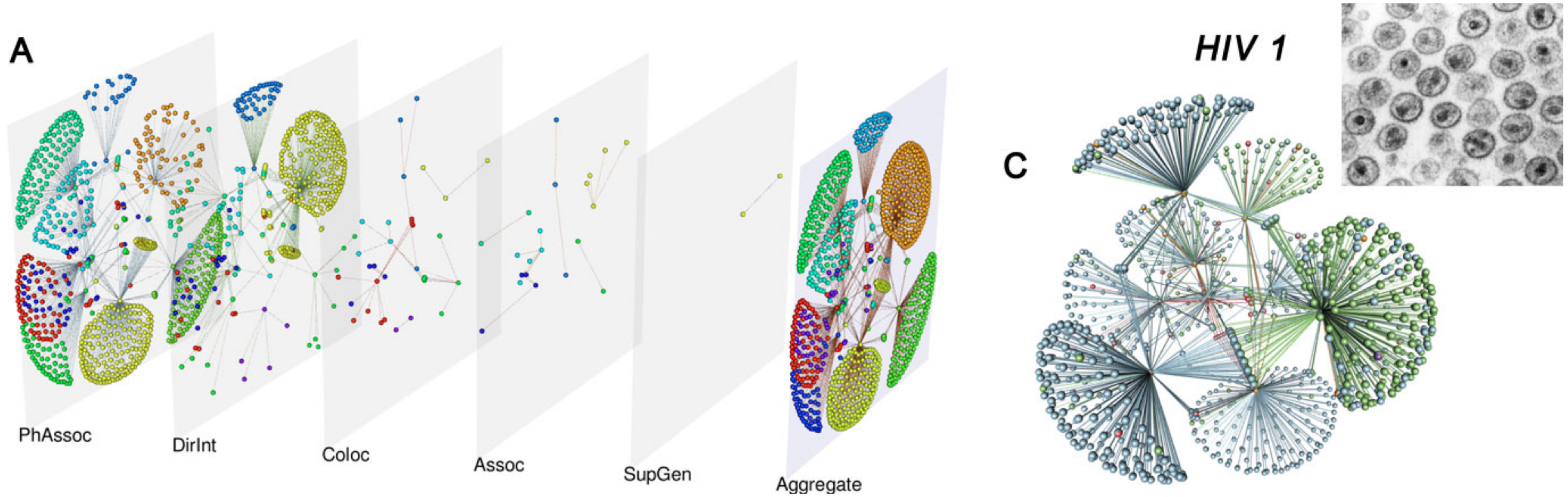


- Multilayer analysis of HIV-1 genetic interaction network

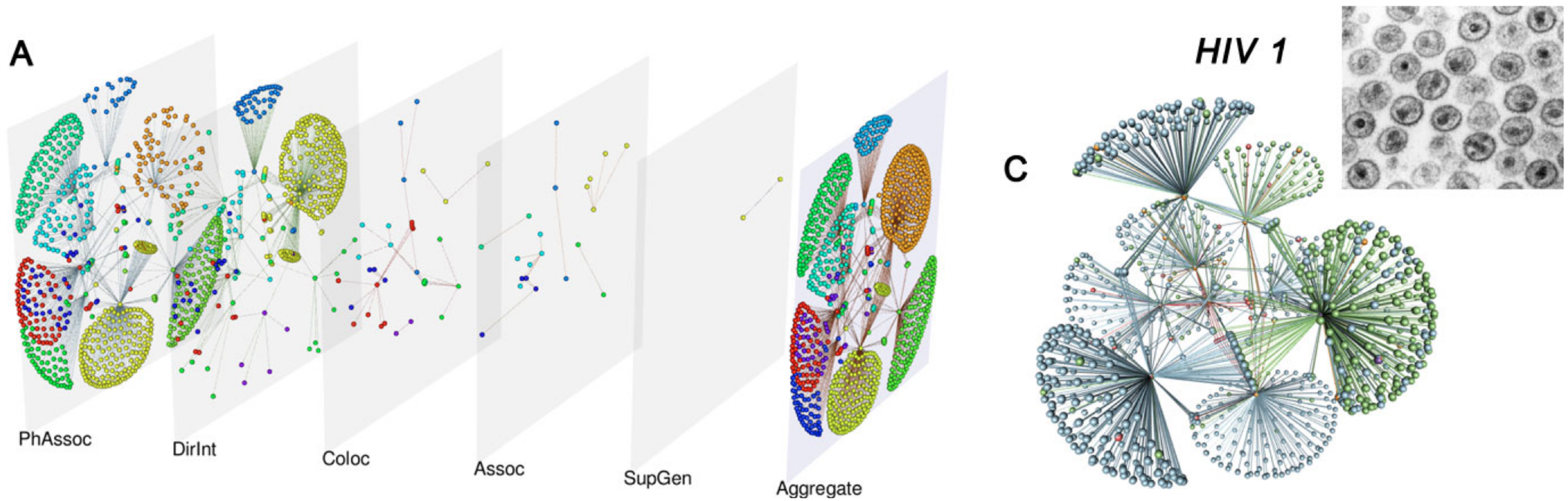
MuxViz: A Tool for Multilayer Analysis and Visualization of Networks, De Do,enico et al, 2015.

Generating 2.5D representations



- If it is essential that same node has exactly the same position over the layers

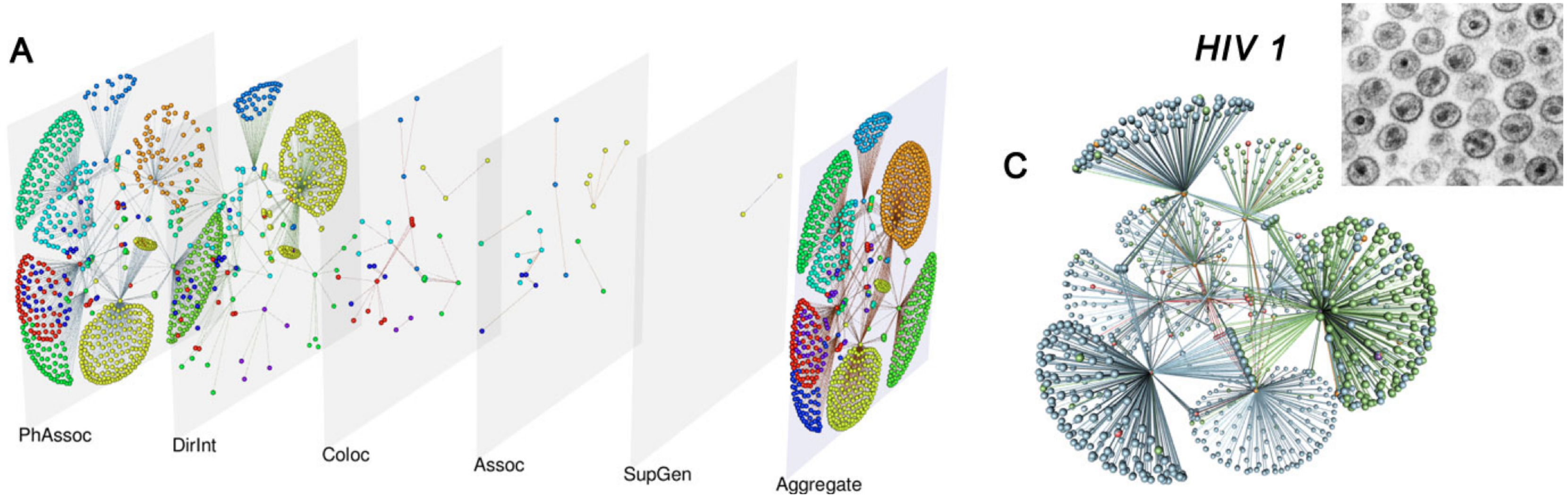
Generating 2.5D representations



- If it is essential that same node has exactly the same position over the layers

How to construct this representation?

Generating 2.5D representations



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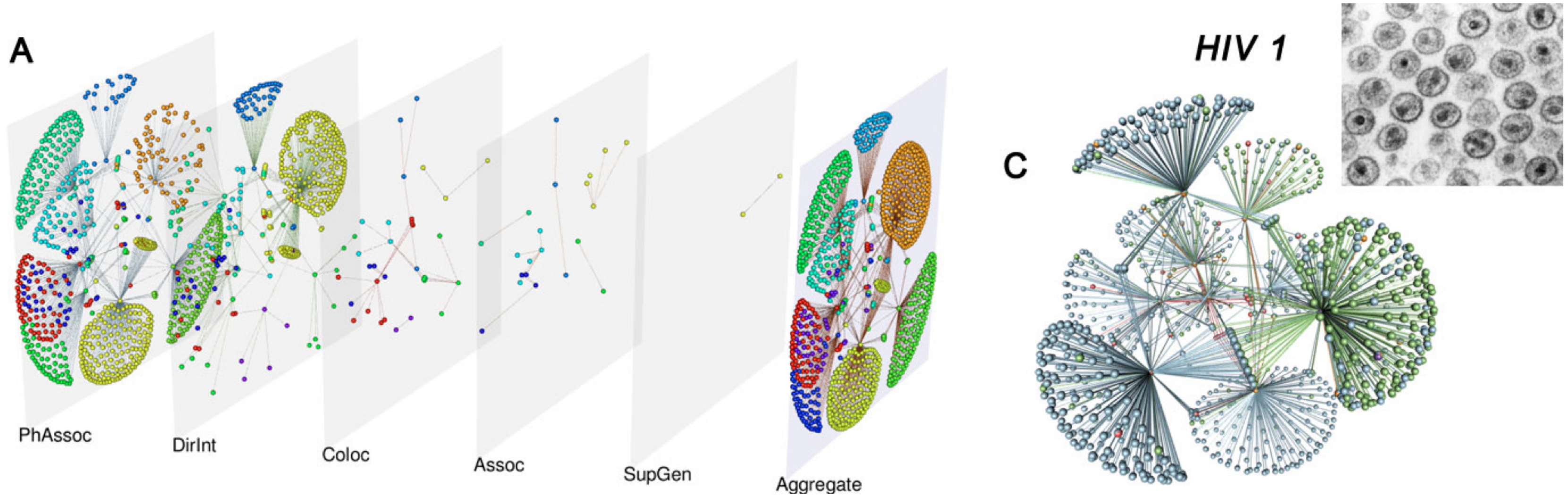
- Aggregate the graphs over the layers into a single graph $G = (V, E)$

Generating 2.5D representations



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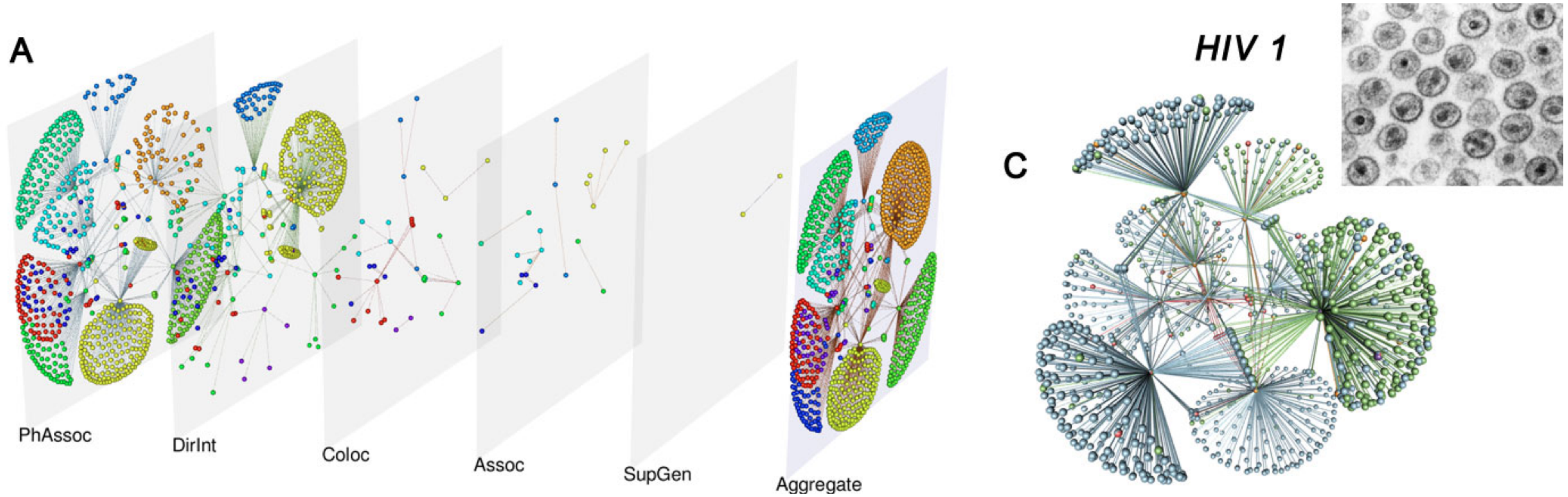
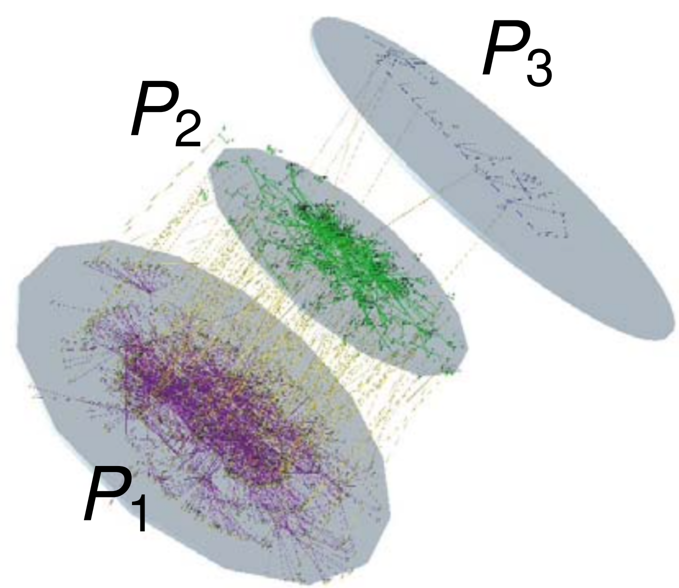


- If it is essential that same node has exactly the same position over the layers
- Aggregate the graphs over the layers into a single graph $G = (V, E)$
- Layout G with a favorite layout method – aggregated layer
- Use coordinates of the nodes of G to construct the layouts of the rest layers

Generating 2.5D representations Fung et al, 2009

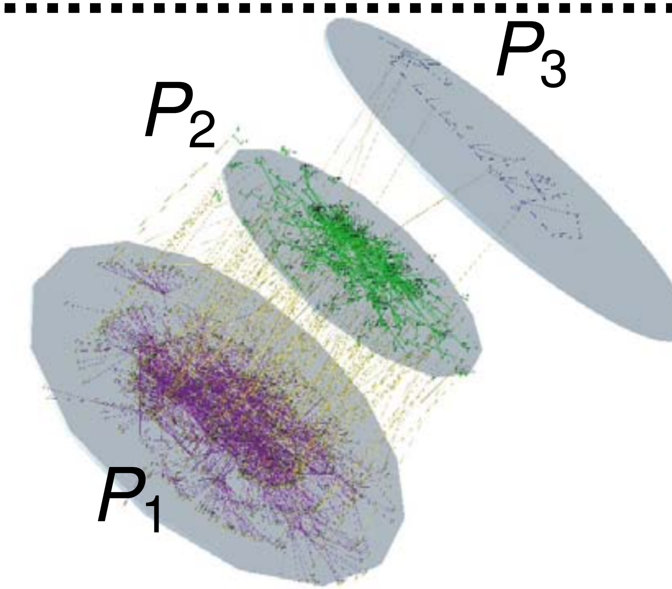
- If it is not essential that same node has exactly the same position over the layers, or there are not many identical nodes over the layers



Generating 2.5D representations Fung et al, 2009

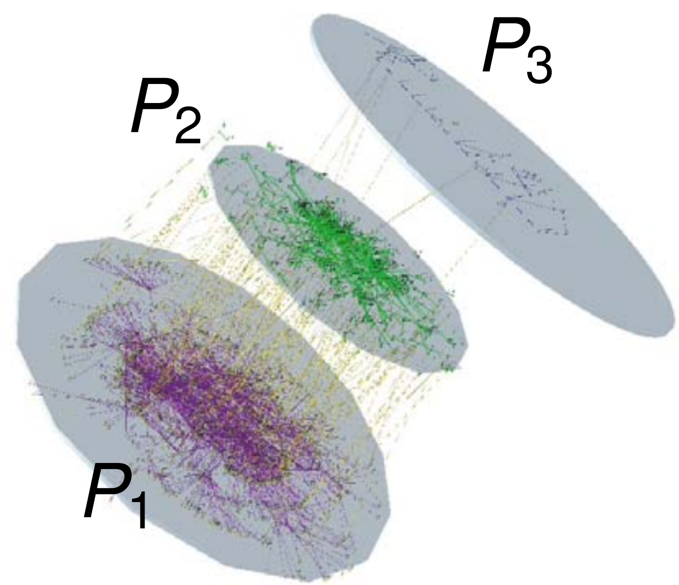
- If it is not essential that same node has exactly the same position over the layers, or there are not many identical nodes over the layers

Same method as before is possible, but how to use the flexibility in node position in order to construct better layout?



Generating 2.5D representations Fung et al, 2009

- If it is not essential that same node has exactly the same position over the layers, or there are not many identical nodes over the layers
- Assume we have 3 layers ℓ_1, ℓ_2, ℓ_3, let G_i be graph induced by $\{(v, \ell_i) \in V_m : v \in V\}$, $i = 1, 2, 3$



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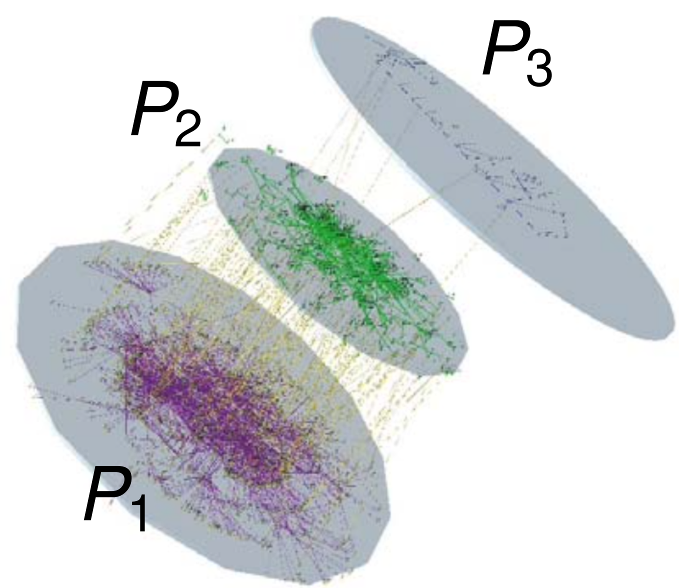

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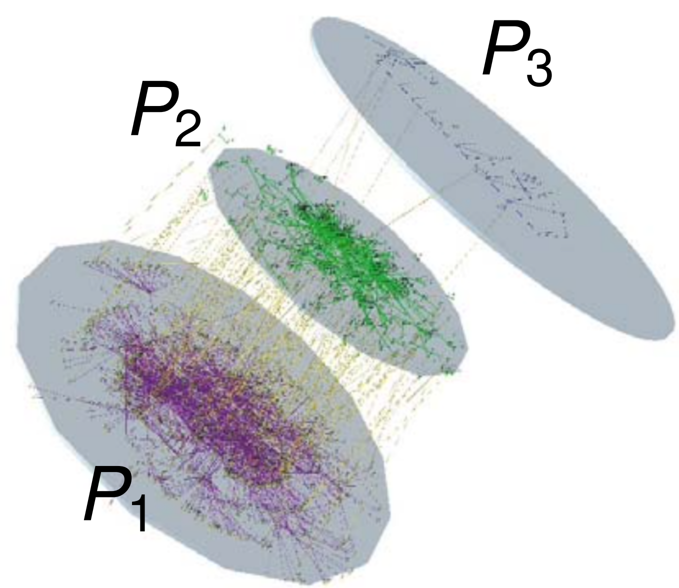
- Assign initial position to each node $(v, \ell_2) \in G_2$ using barycenter of (v, ℓ_1) and (v, ℓ_3)



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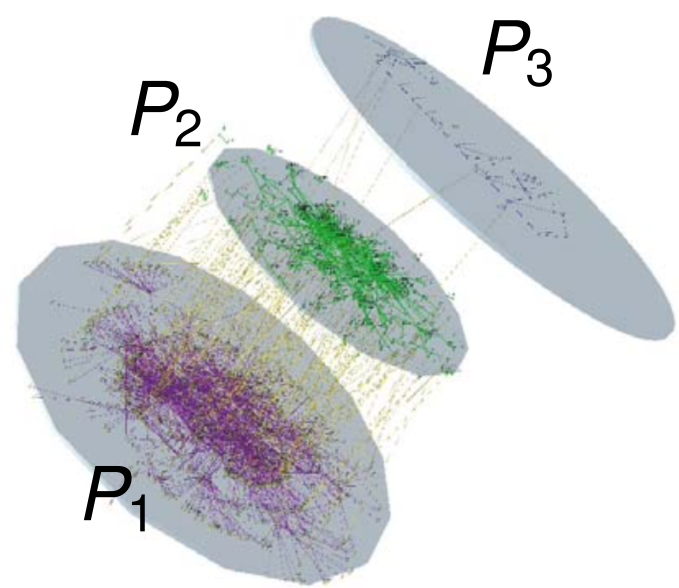

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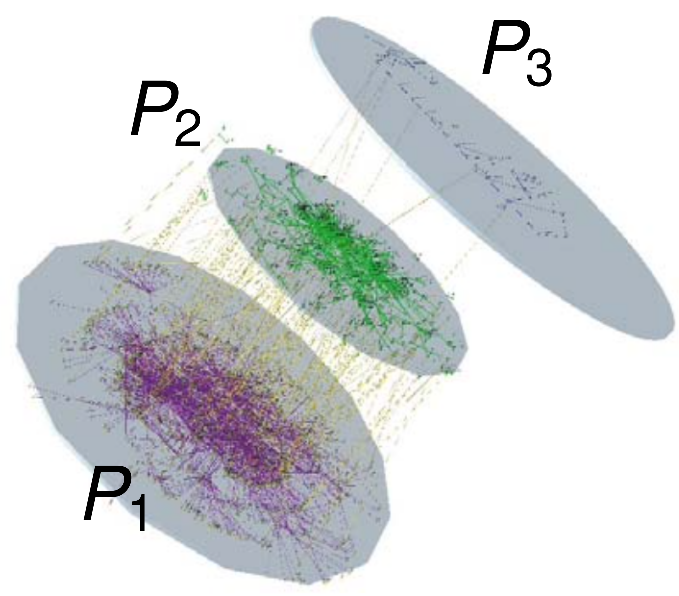
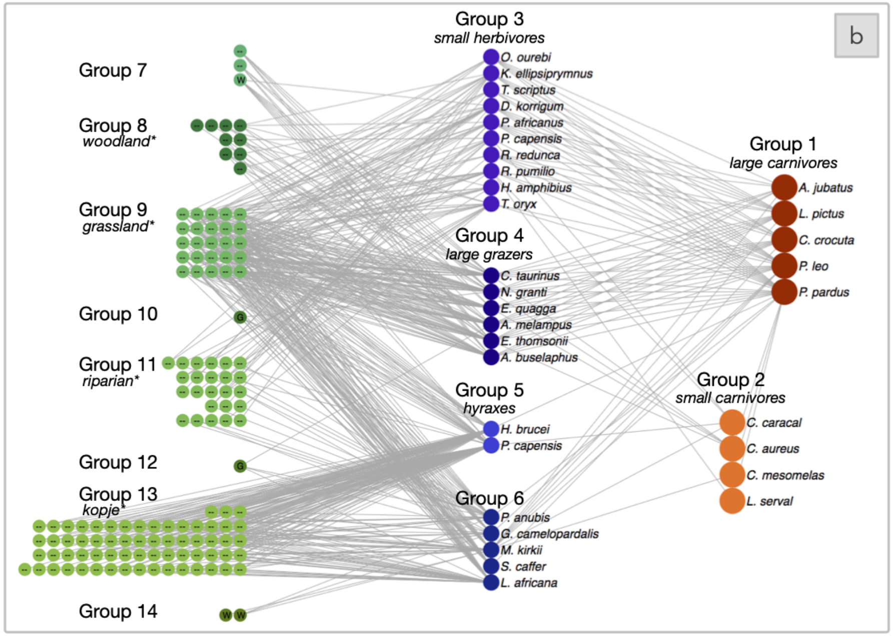
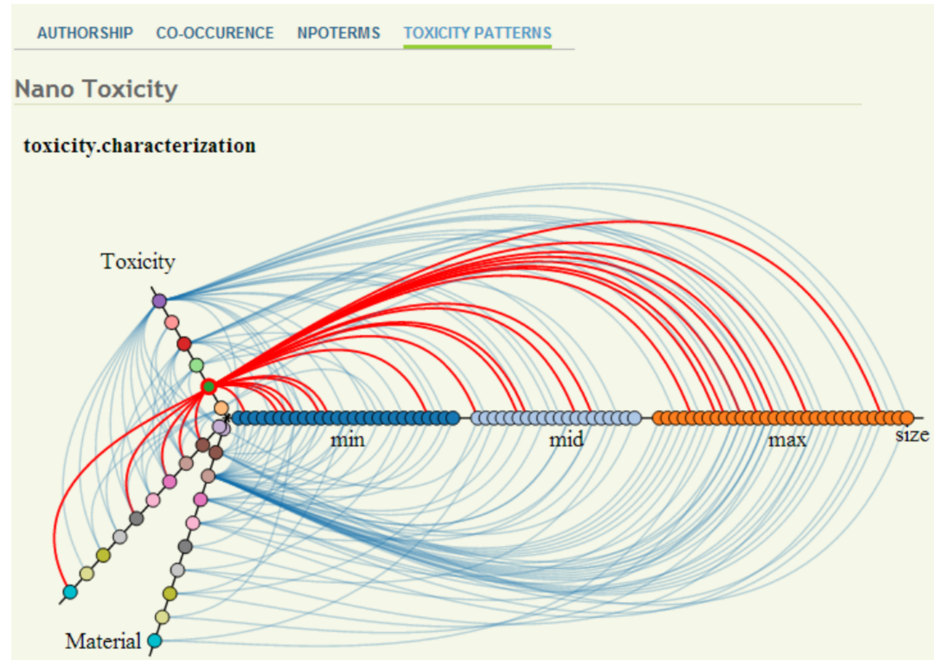
- Model inter-layer edges as zero-length spring (attraction only)
- Draw G_2 and the inter-layer edges using a force directed layout



Edge clutter in multilayer visualizations





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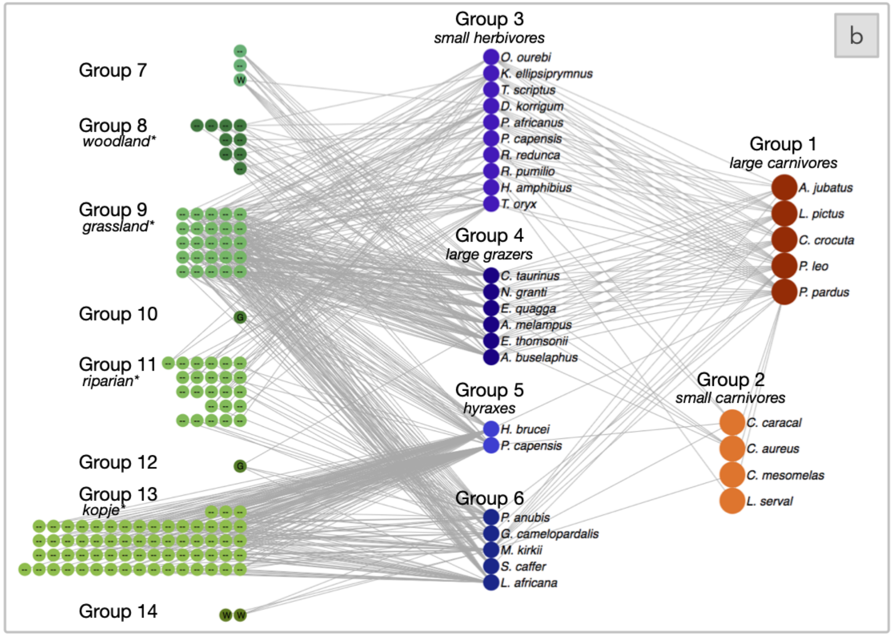
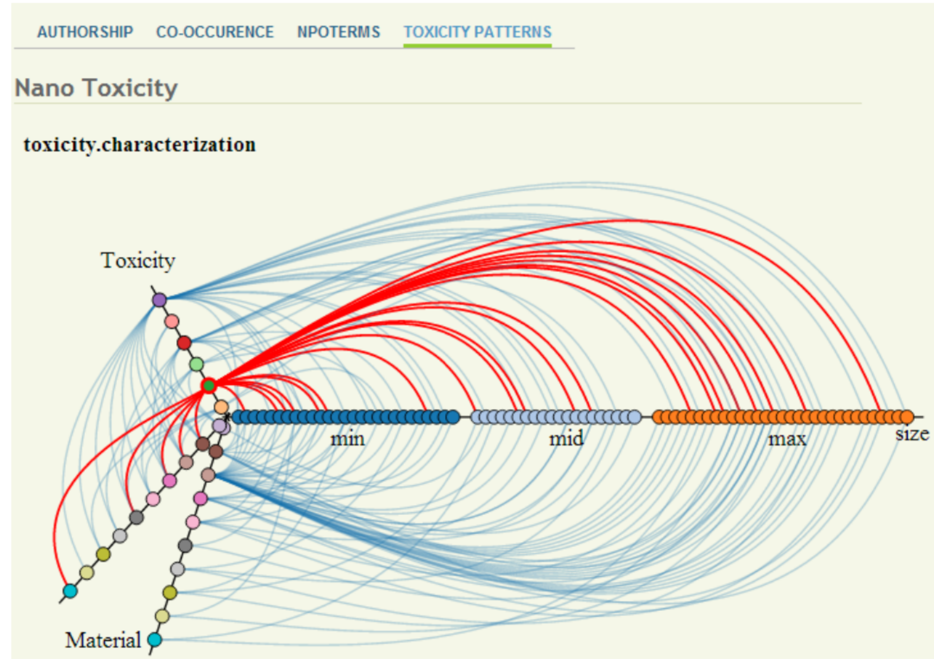
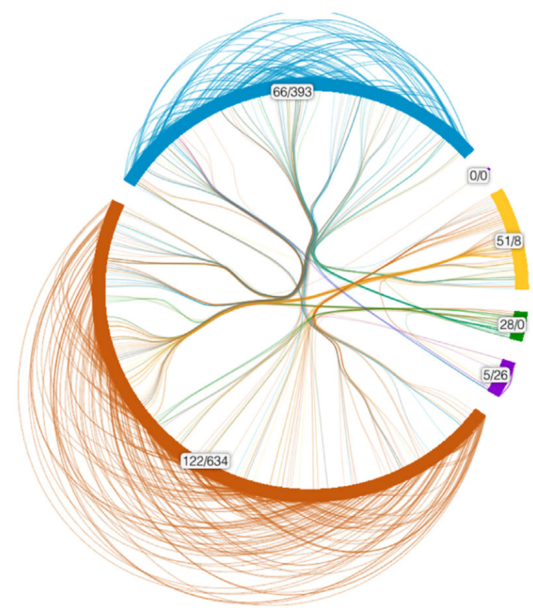


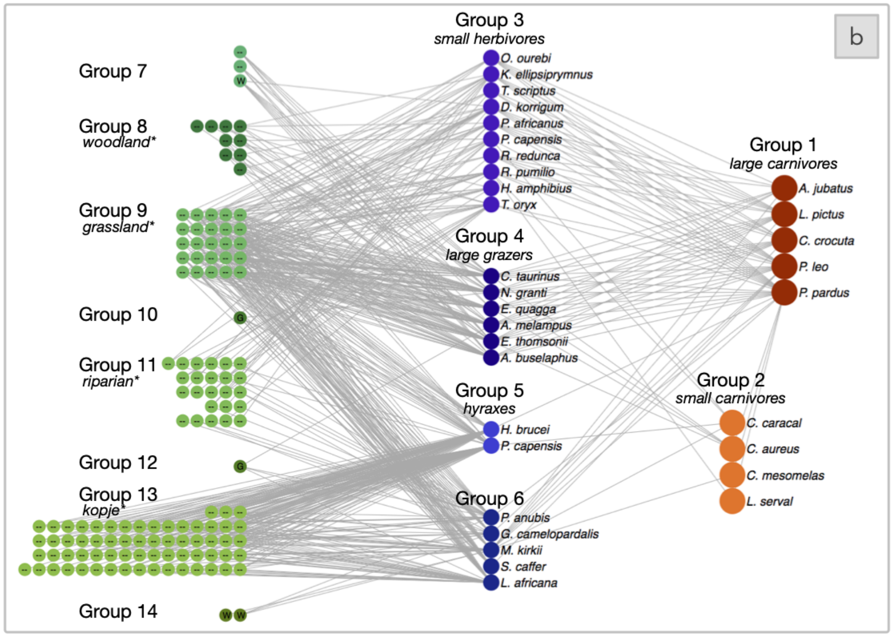


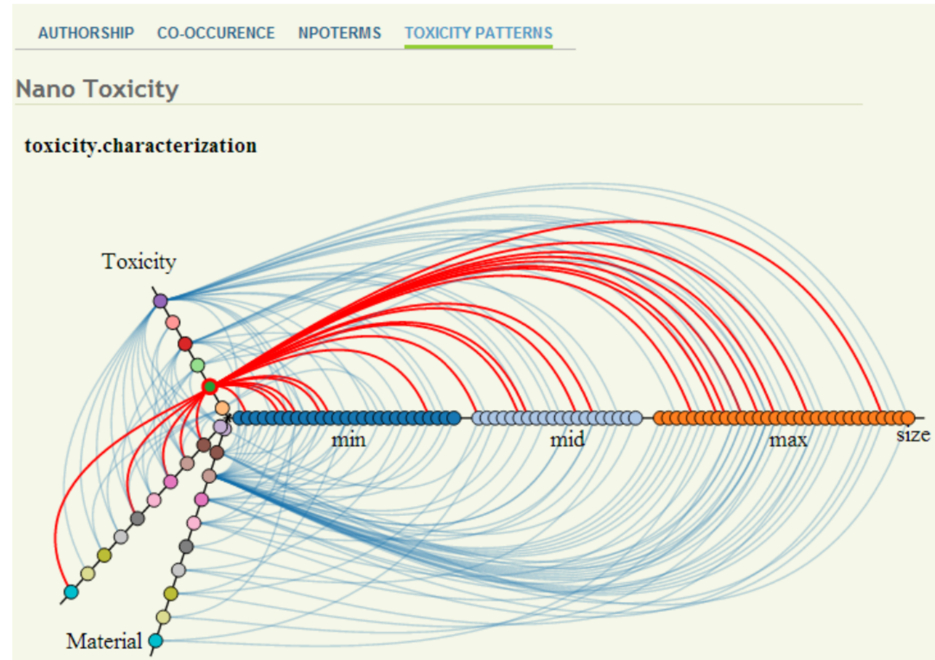
- edge bundling as a method to layout edges in multilayer network visualizations



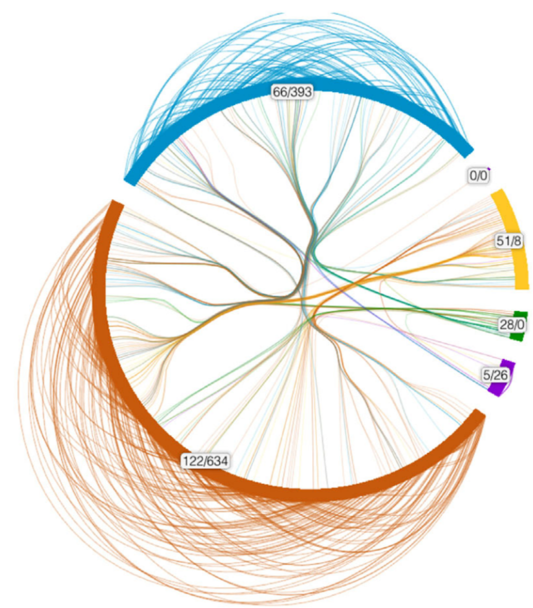
Visualization techniques for categorical analysis of social networks with multiple edge sets, Crnovrsanin et al, 2014

Edge clutter in multilayer visualizations





- edge bundling as a method to layout edges in multilayer network visualizations
- bundle only the inter-layer (or intra-layer) edges



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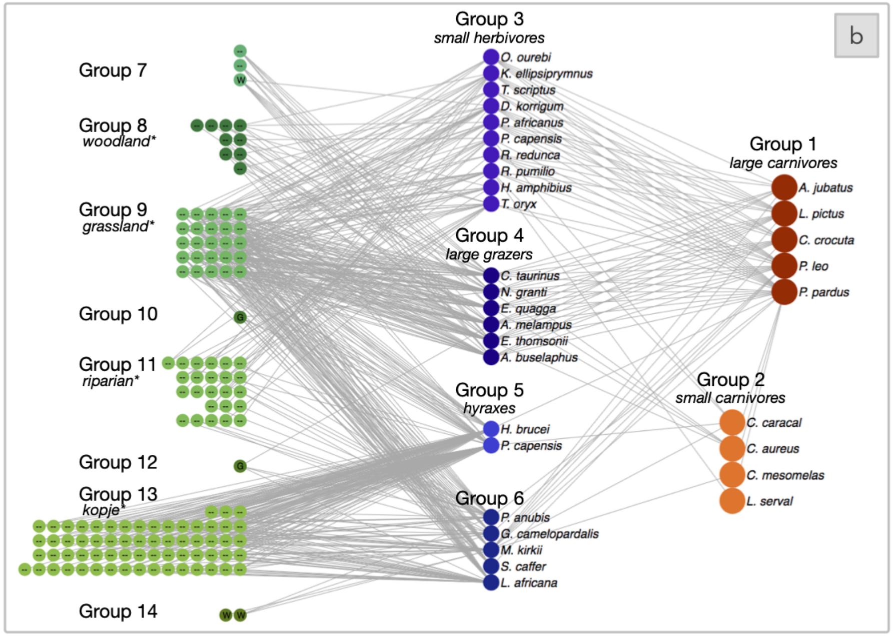
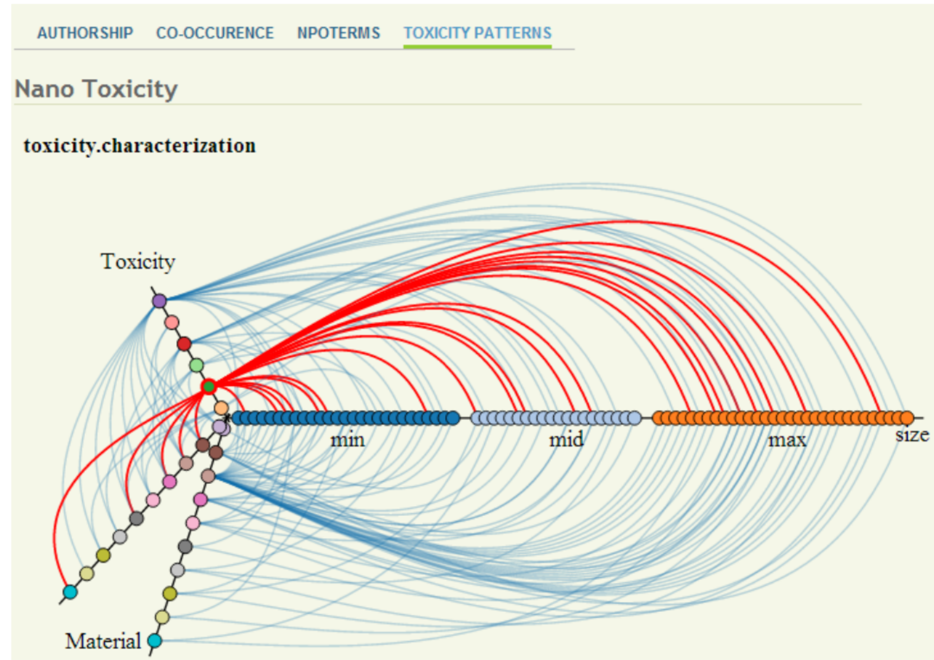
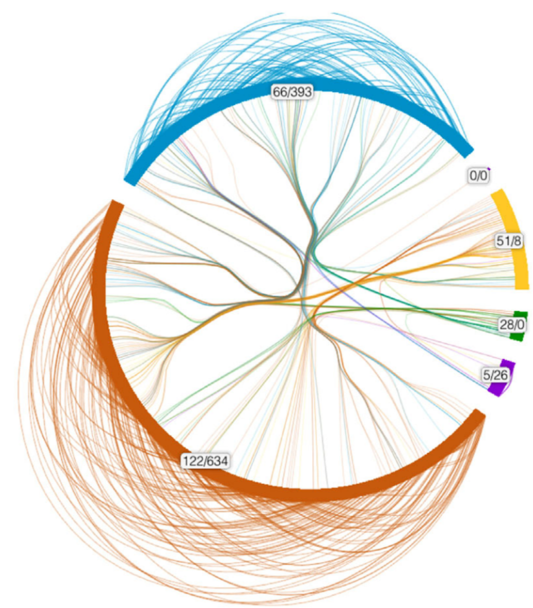




- edge bundling as a method to layout edges in multilayer network visualizations
- bundle only the inter-layer (or intra-layer) edges
- edge bundling is not specific for multilayer network visualizations



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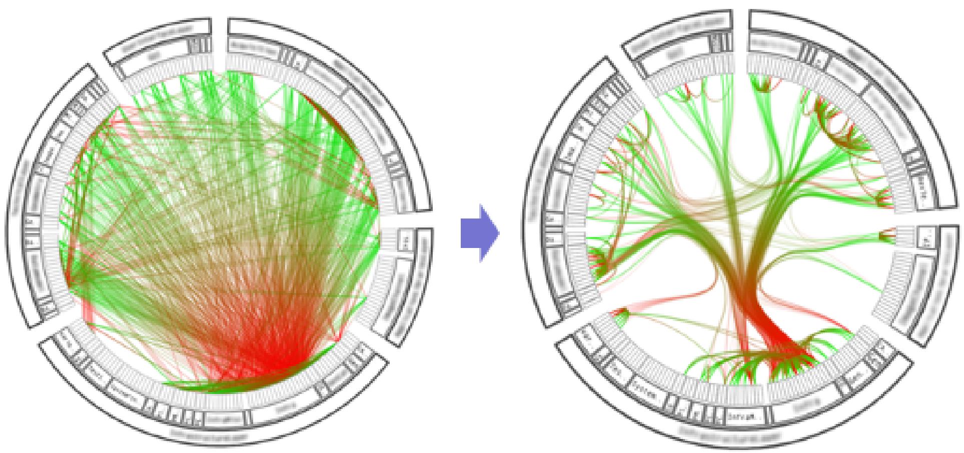
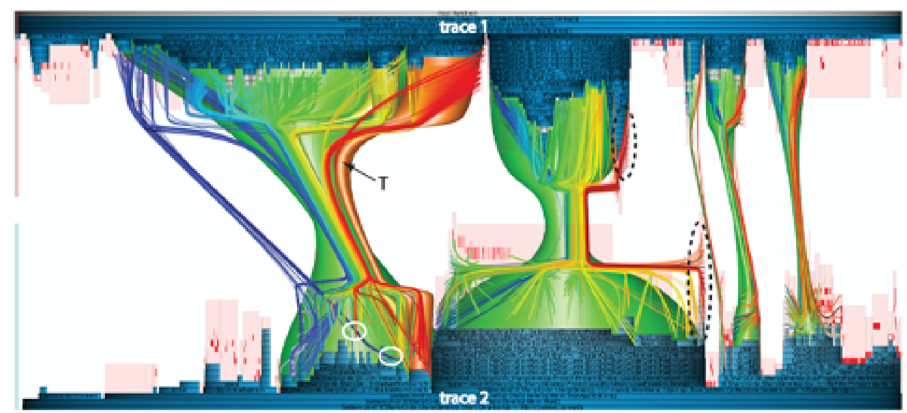
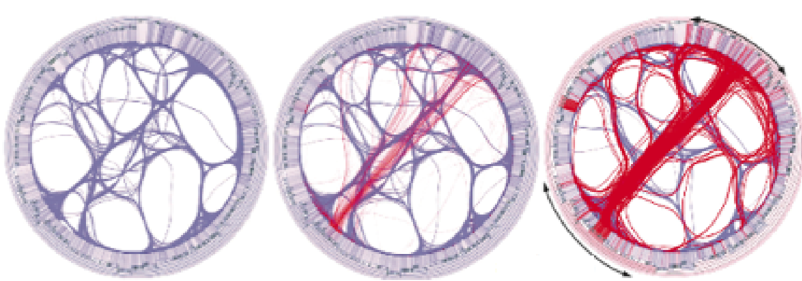
Edge bundling

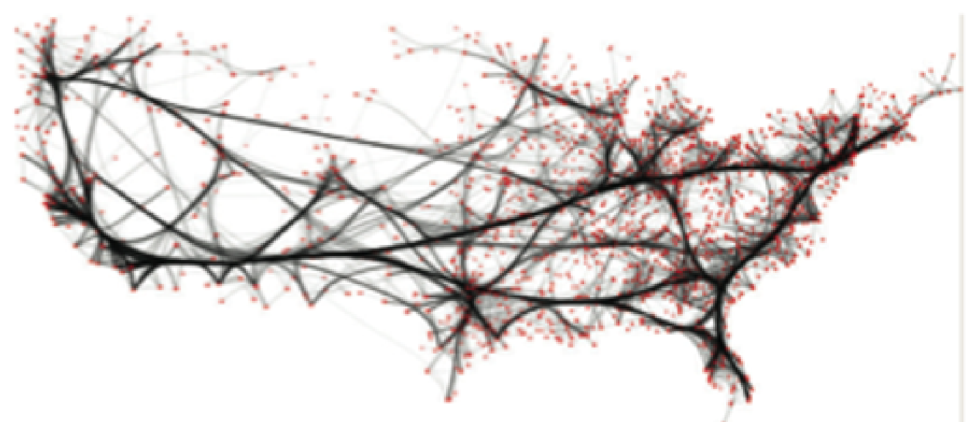
Method for reduction of clutter in a graph layout

“Change the shape of edges by visually bundling them together analogous to the way electrical wires and network cables are merged into bundles...” [Holten, van Wijk, 09]

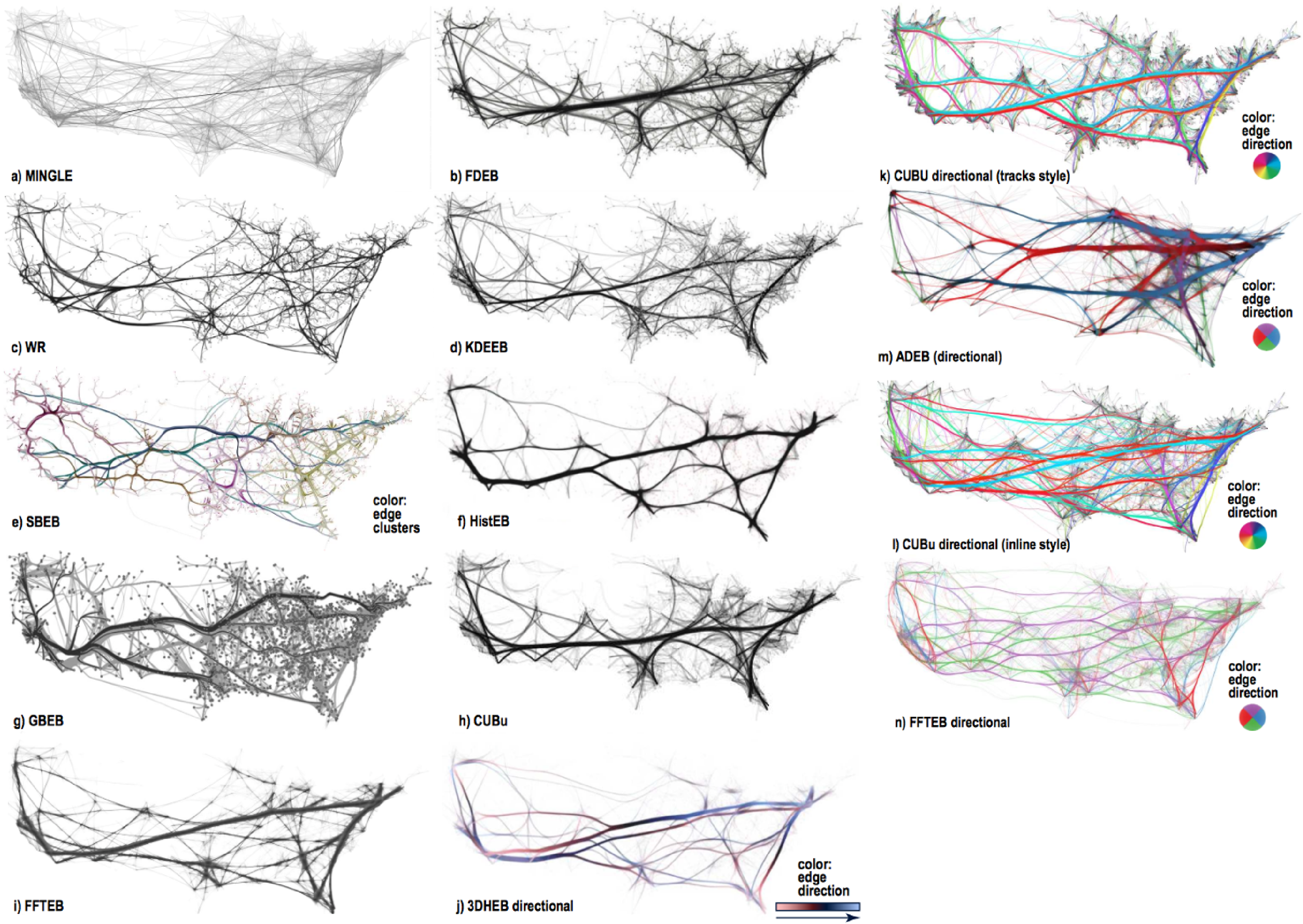




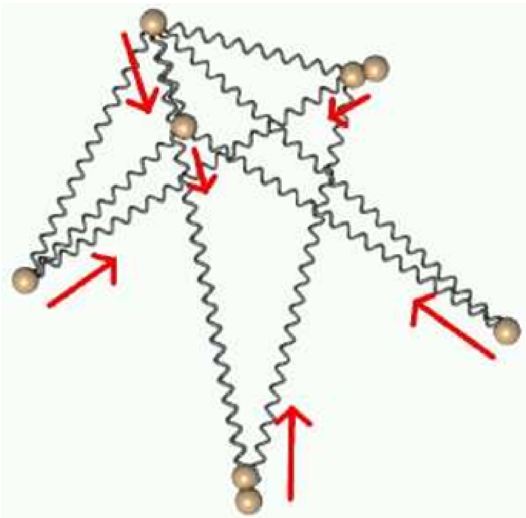




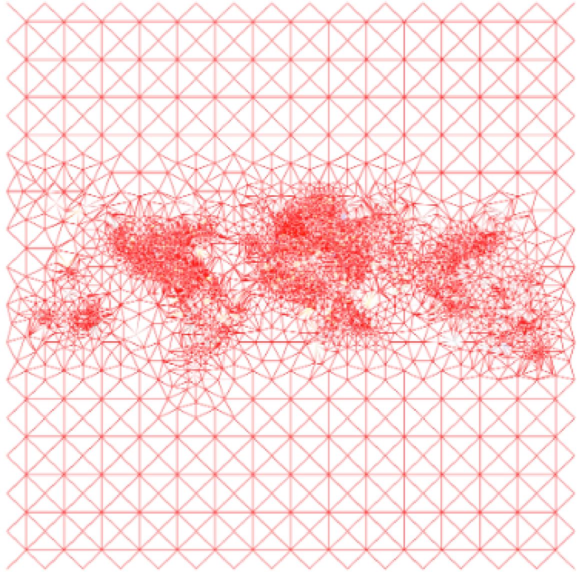
Many methods



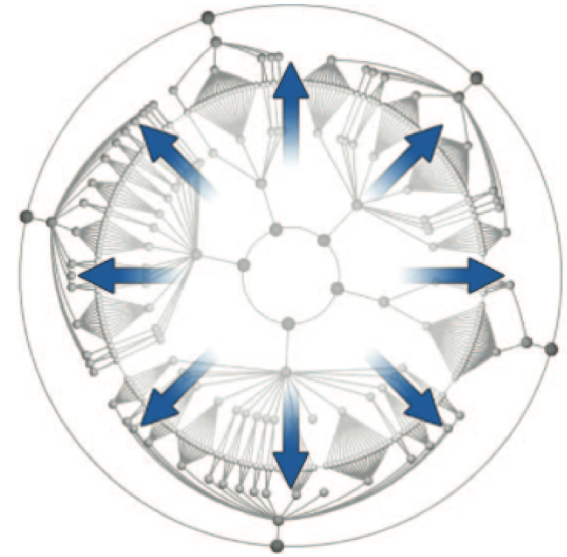
Multiple techniques



spring embedders (FDEB)

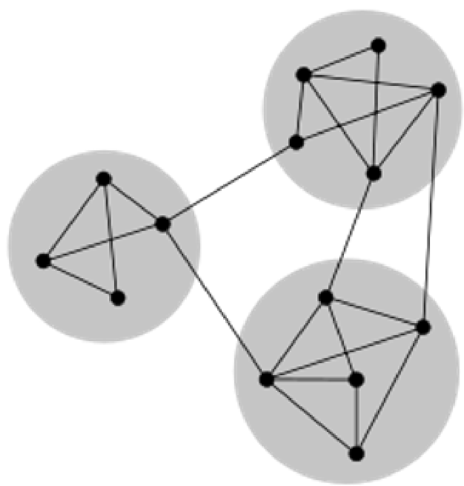


Voronoi/Delaunay diagrams

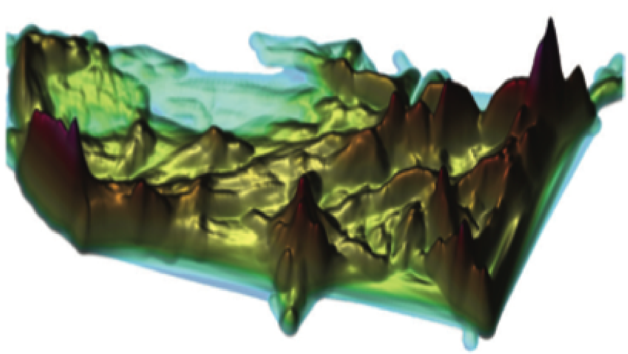


tree layouts & splines

medial axes

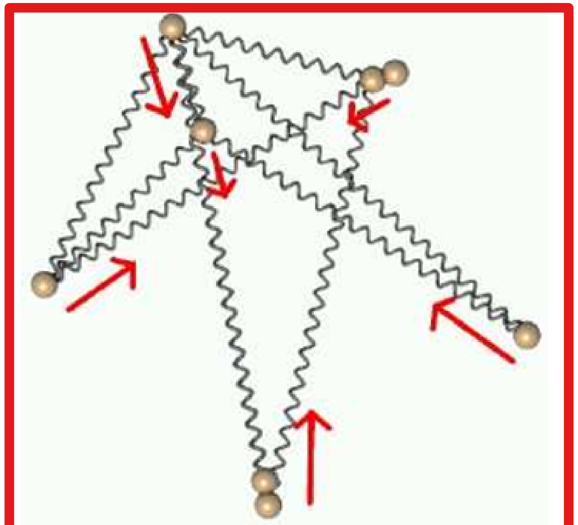


graph clustering



kernel density estimation

Multiple techniques



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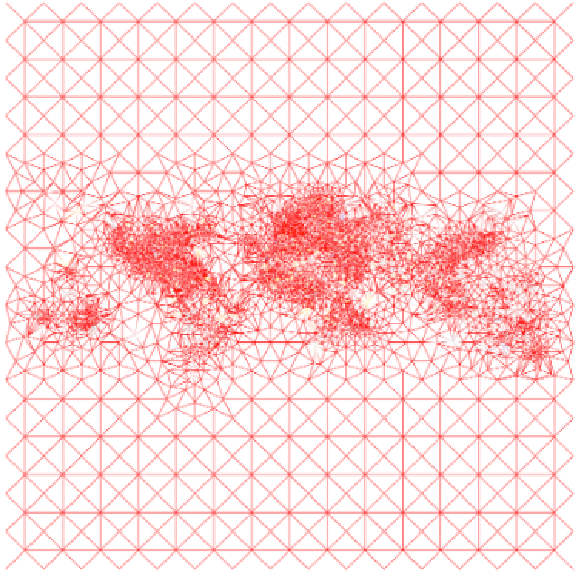

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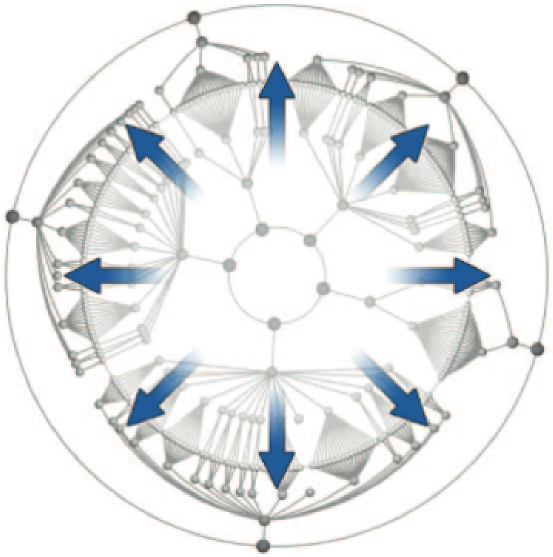

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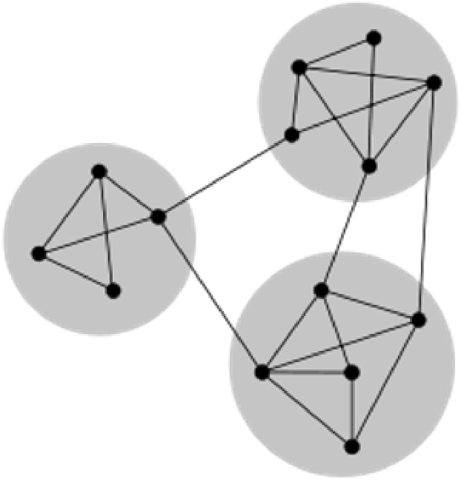

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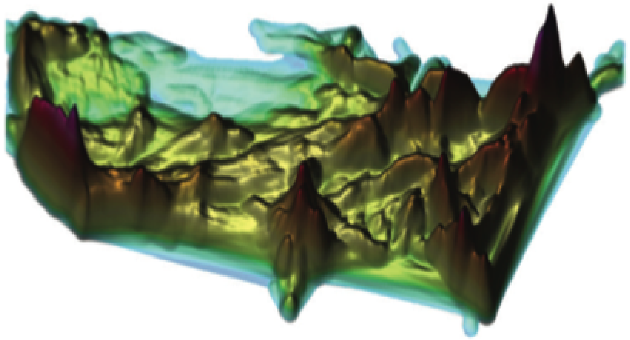
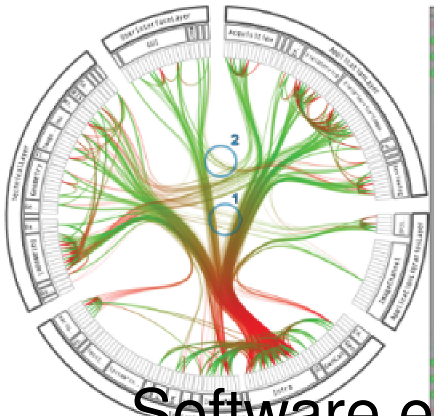
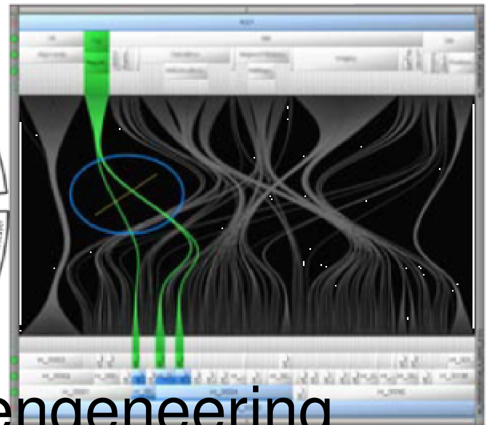


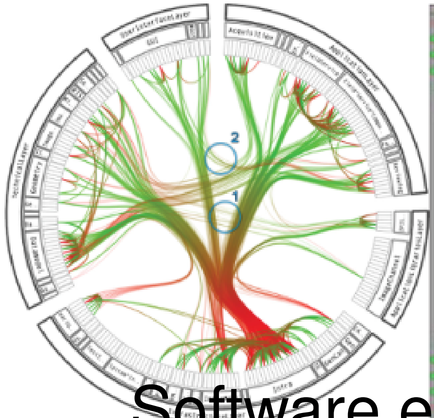
kernel density estimation

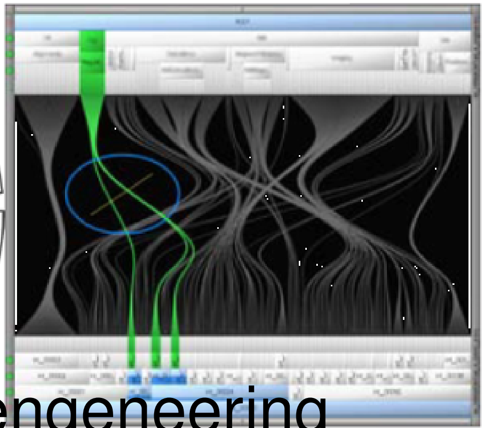
Edge bundling: applications

Software engeneering

Edge bundling: applications





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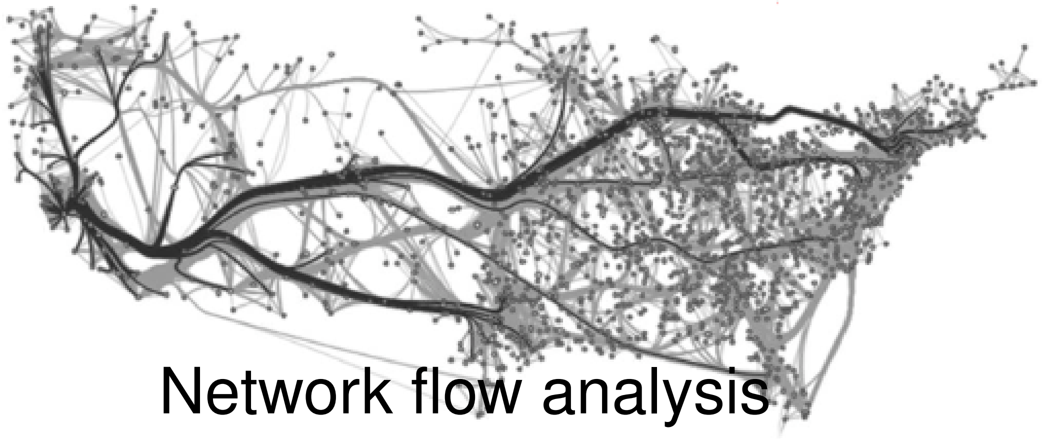
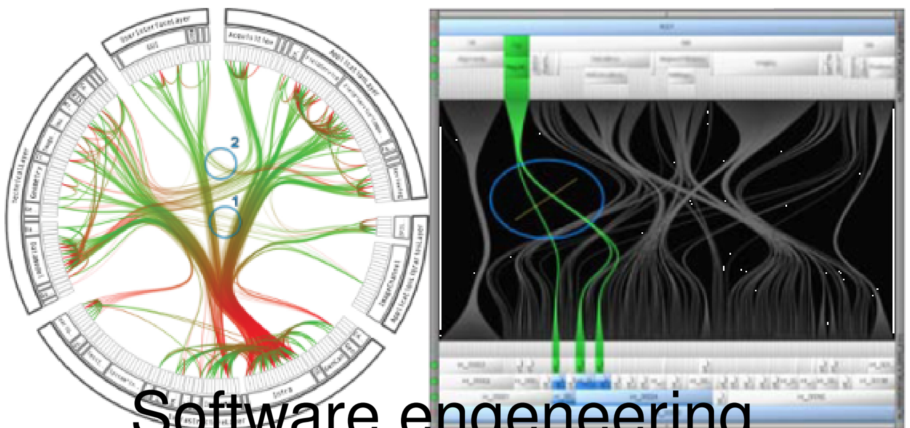


Network flow analysis

Edge bundling: applications



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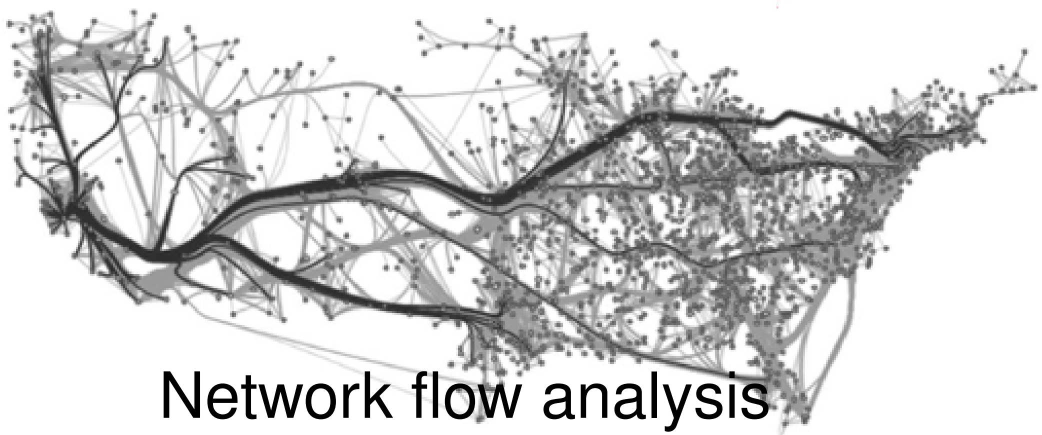
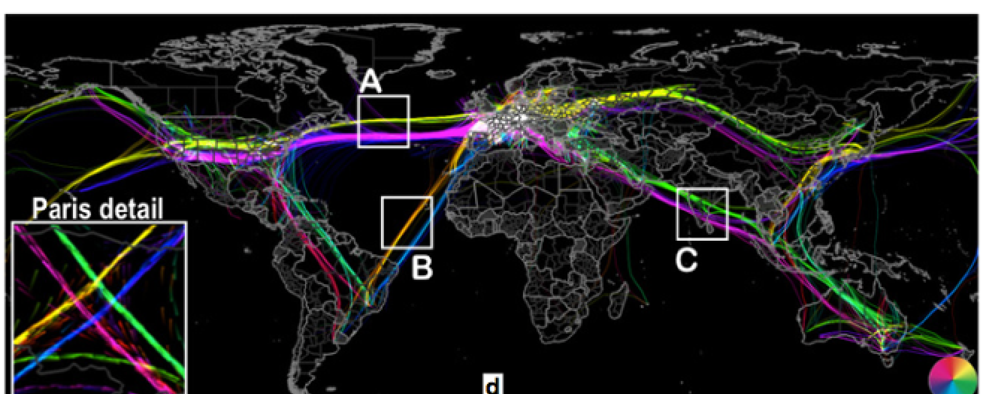
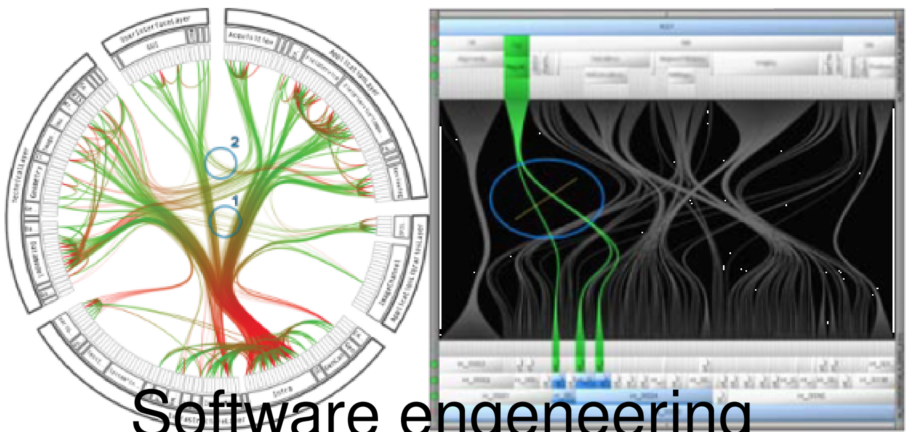


Network flow analysis



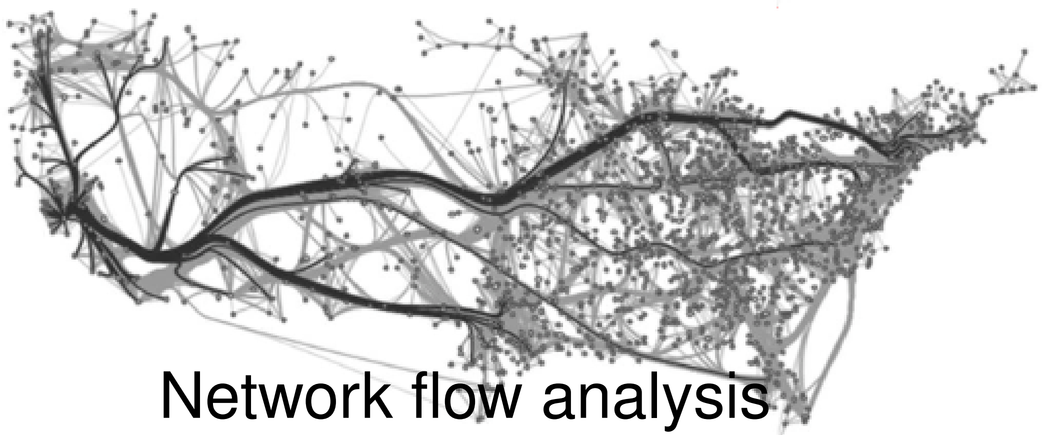
Air traffic control

Edge bundling: applications



Software engeneering

Bioinformatics



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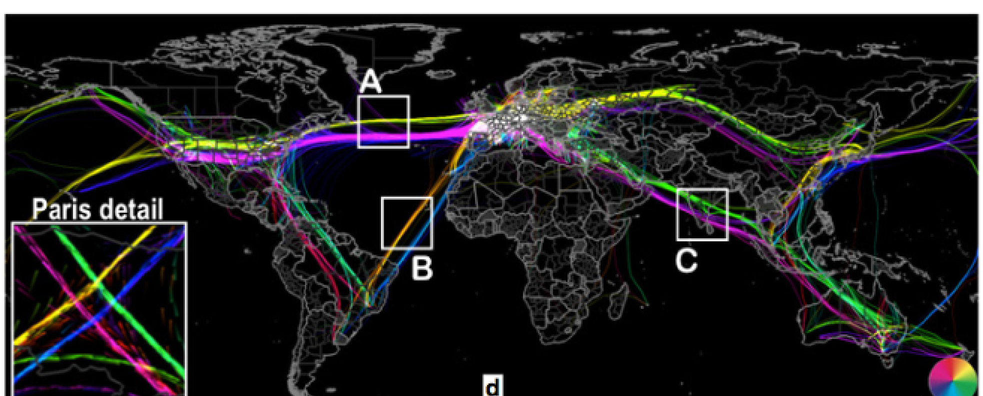

Air traffic control

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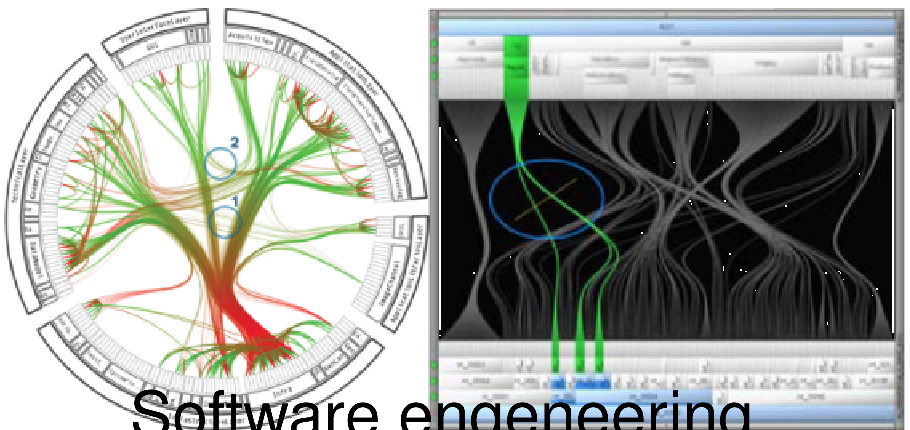

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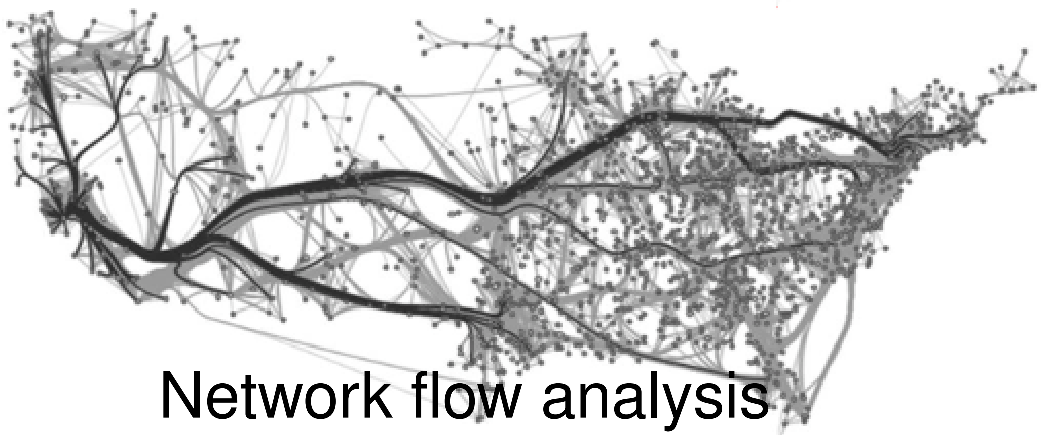

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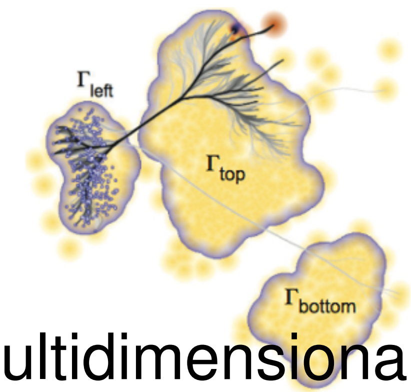
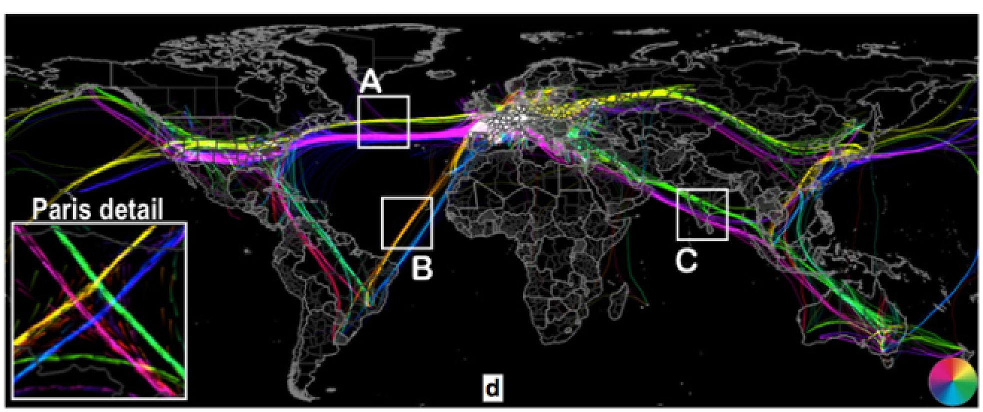


Multidimensional data



Air traffic control

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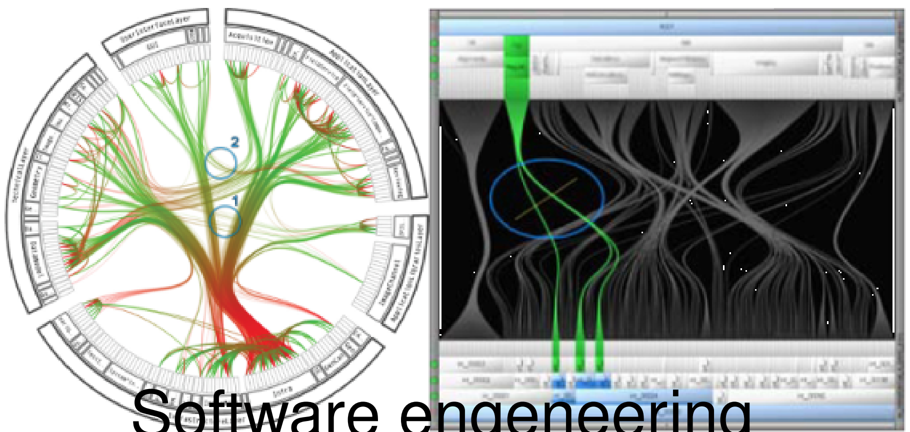

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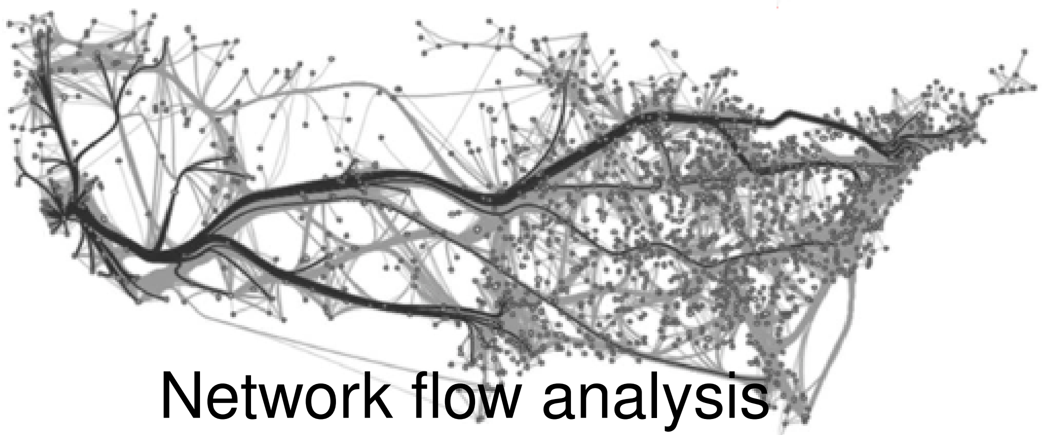

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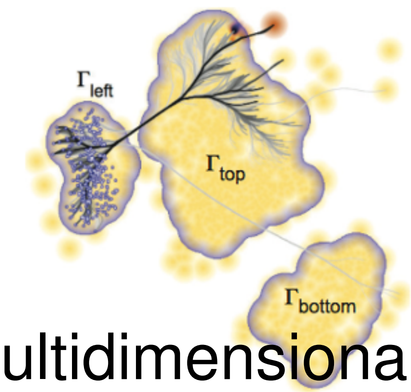

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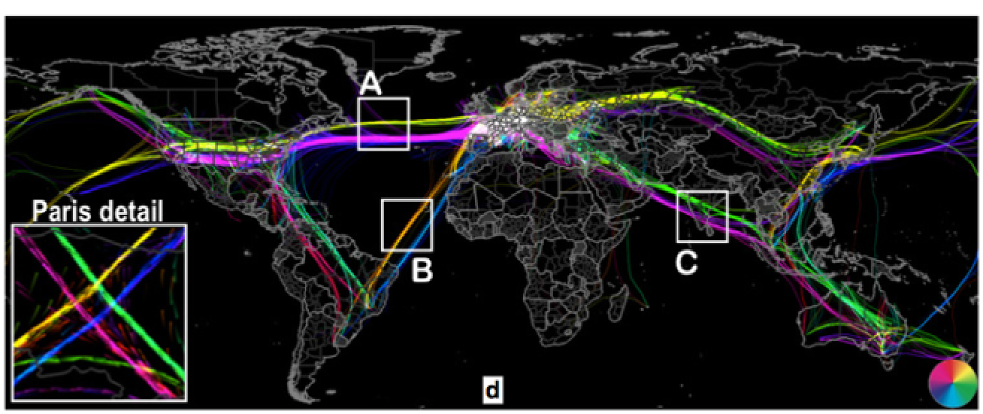

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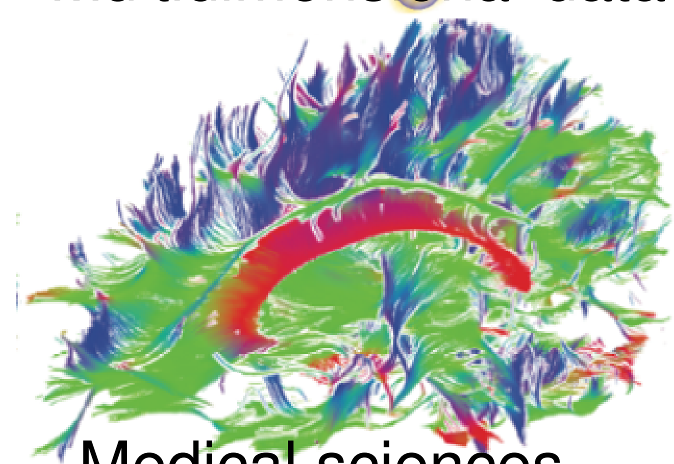
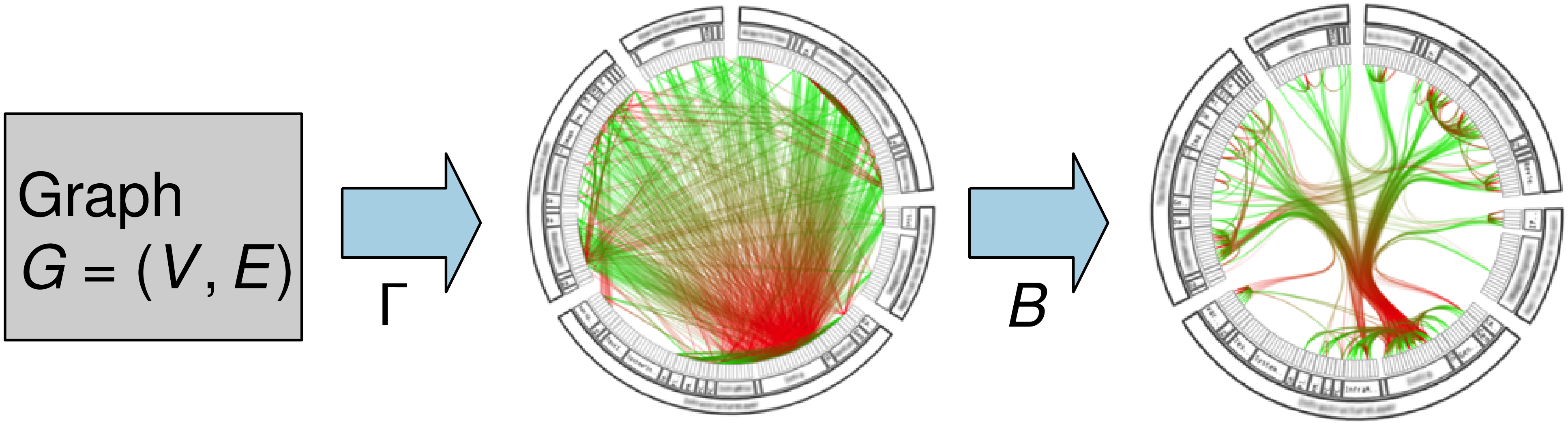


Medical sciences

Edge bundling: definition



Γ – drawing/layout function

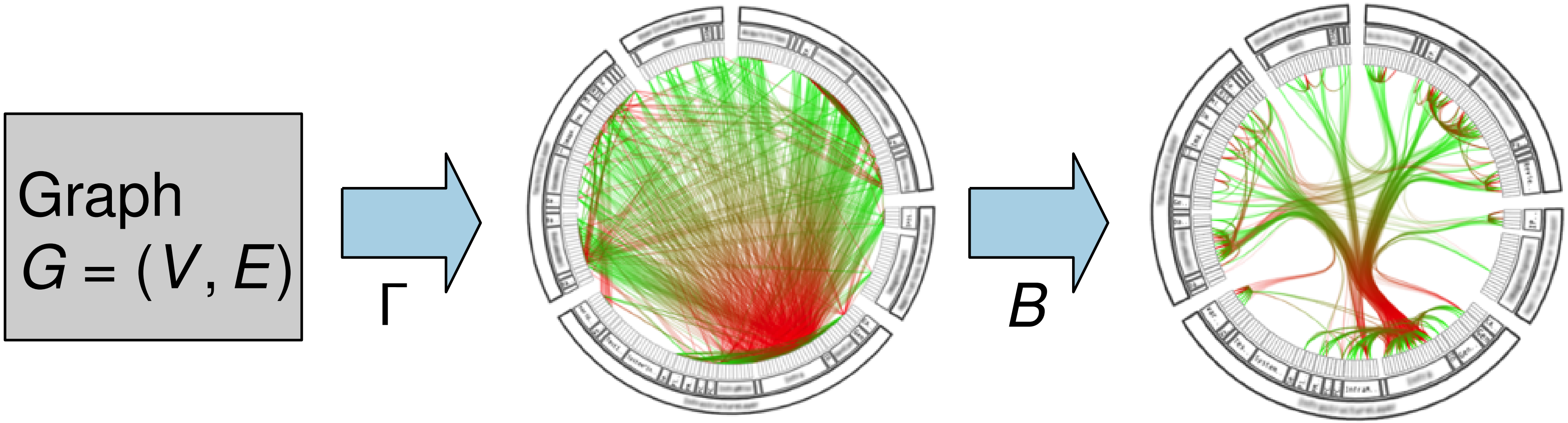
B – bundling function

$$\forall (e_i, e_j) \in E \times E \text{ such that } e_i \neq e_j \wedge k(e_i, e_j) < k_{\max} \rightarrow$$
$$\delta(B(\Gamma(e_i)), B(\Gamma(e_j))) \ll \delta(\Gamma(e_i), \Gamma(e_j))$$

$k_{\max}$ – maximum similarity of the edges that still need to be bundled

k–similarity of two edges; δ – similarity of two curves

Edge bundling: definition



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two edges are similar

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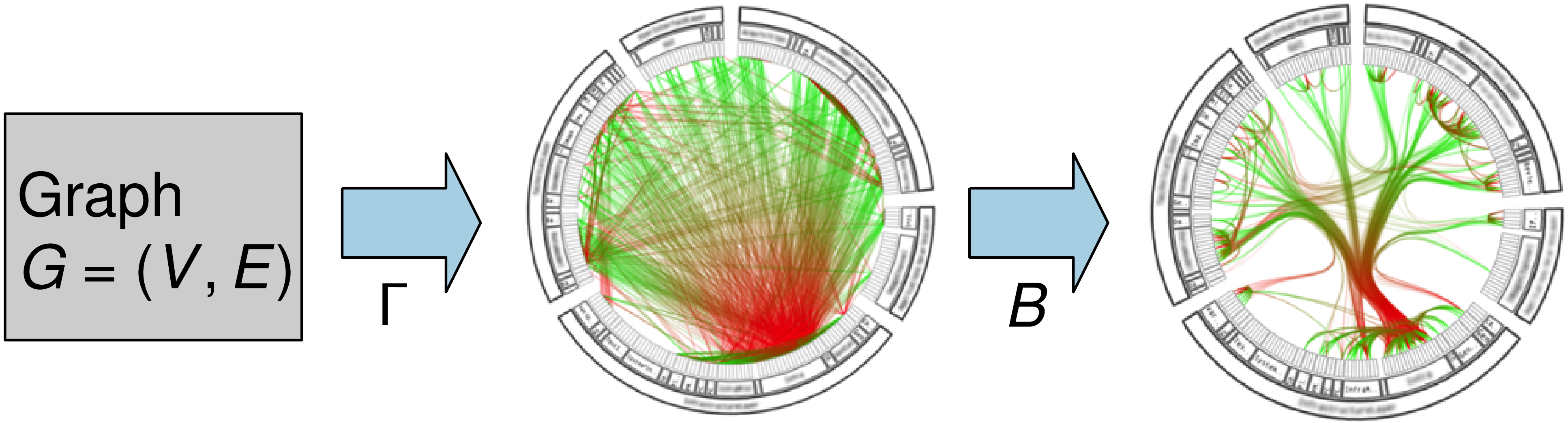

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the distance between curves after bundling is small

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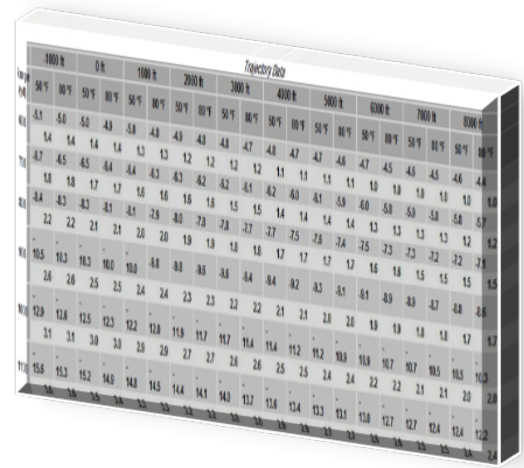

Data-based similarities

- Structured-based
- Attribute-based

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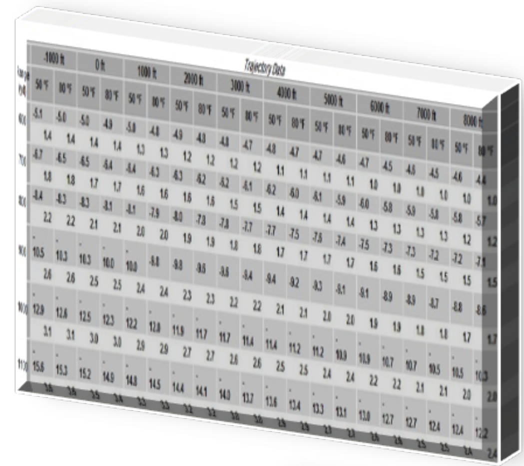

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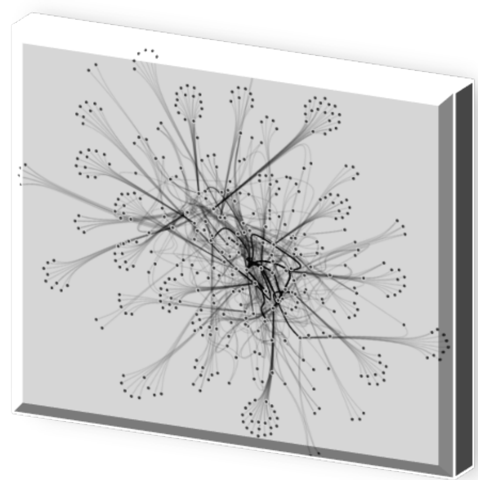
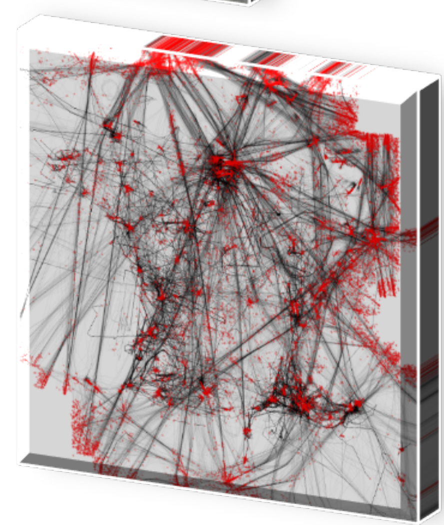
Drawing-based similarities

- Geometric-based
- Image-based





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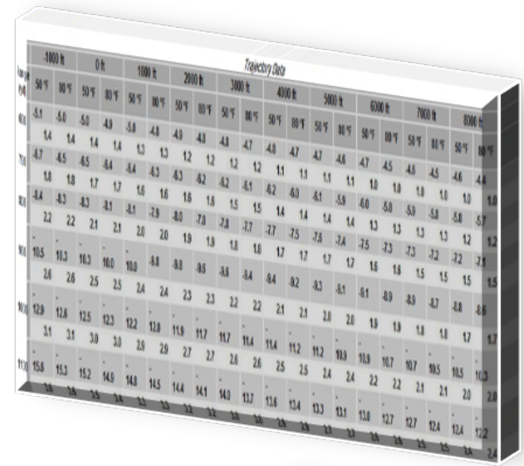

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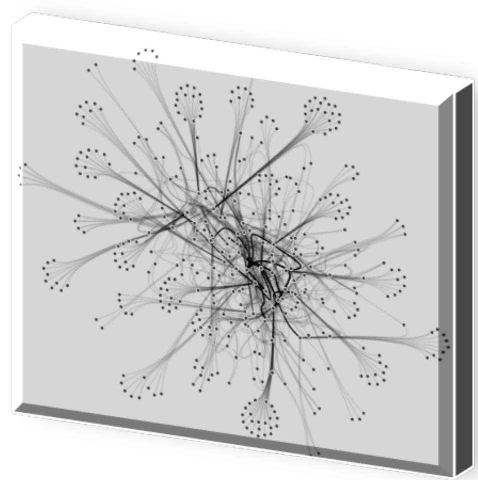
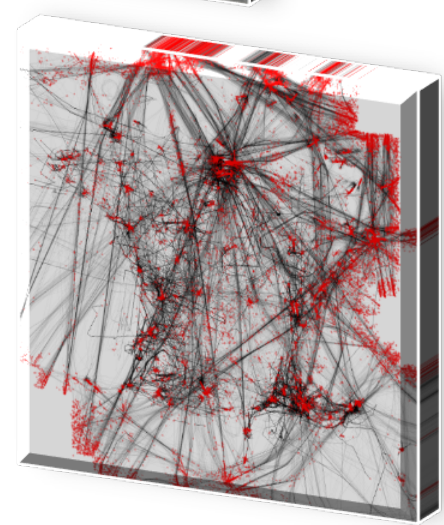
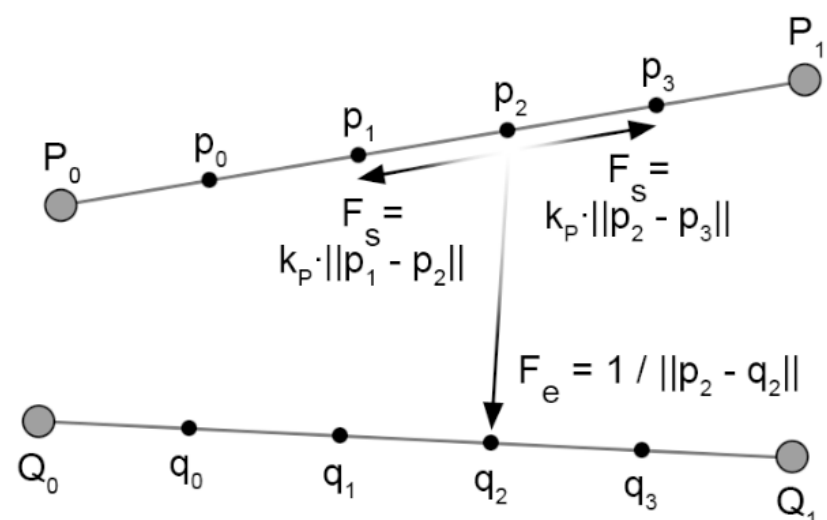




Edge bundling Holten and van Wijk, 2009

- Assume two edges *P* and *Q* need to be bundled (which – later). We say they are *interacting*.



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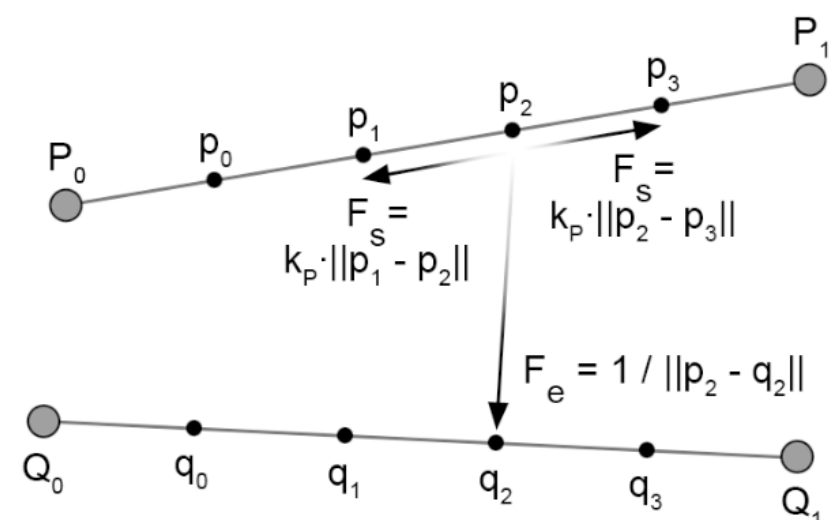

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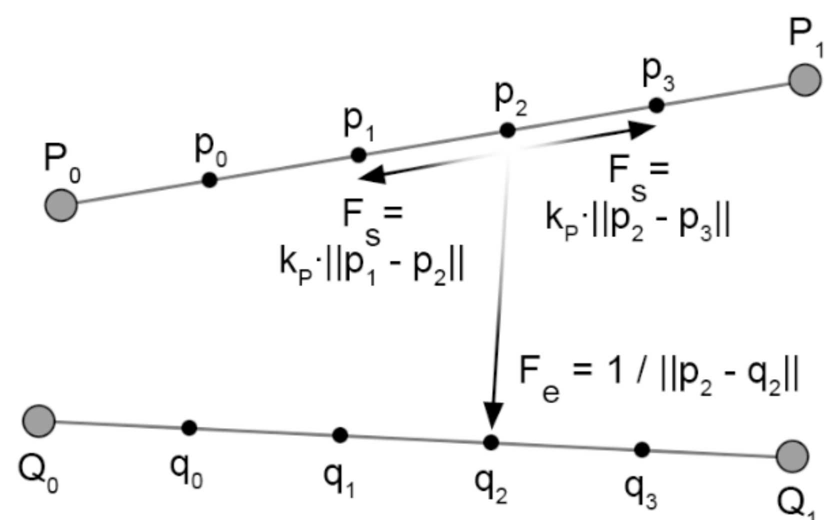

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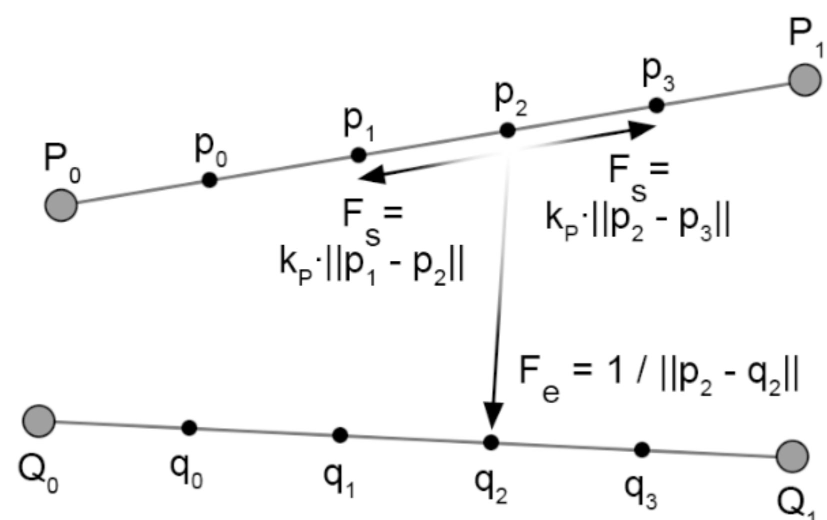

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Spring force

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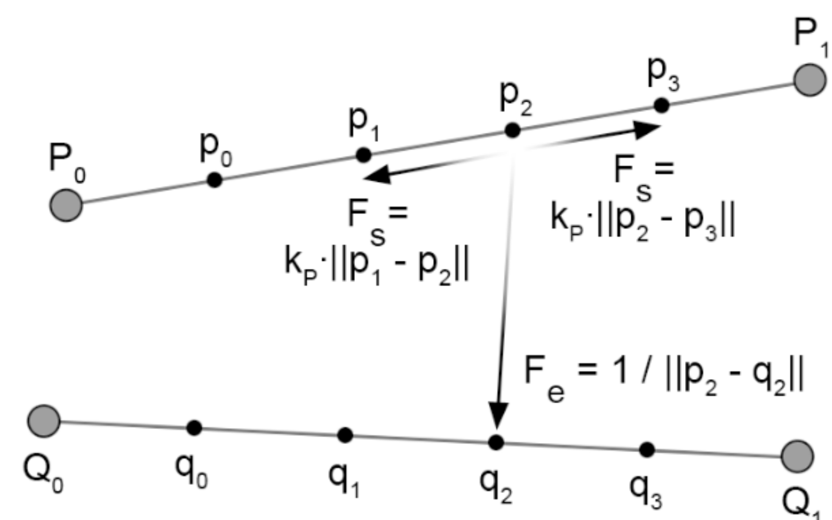

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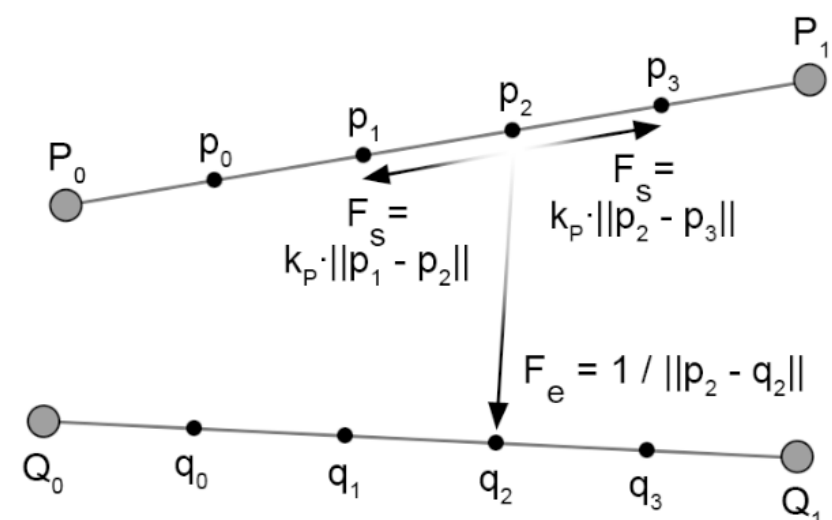

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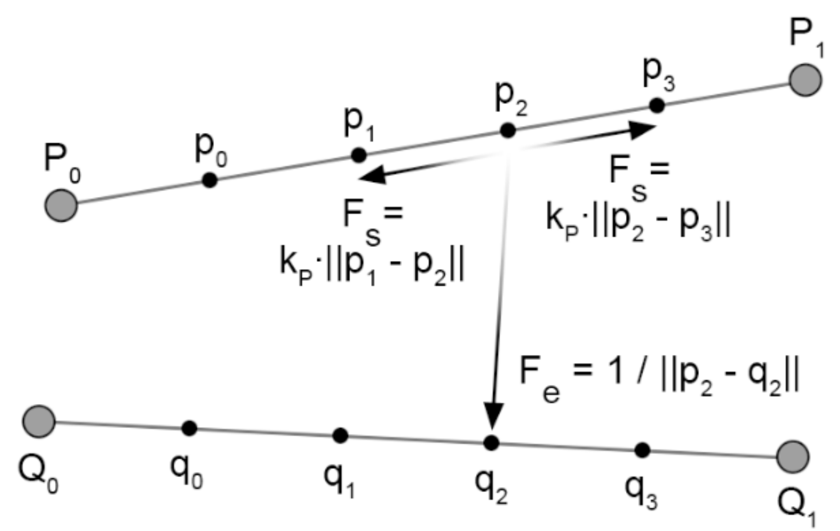

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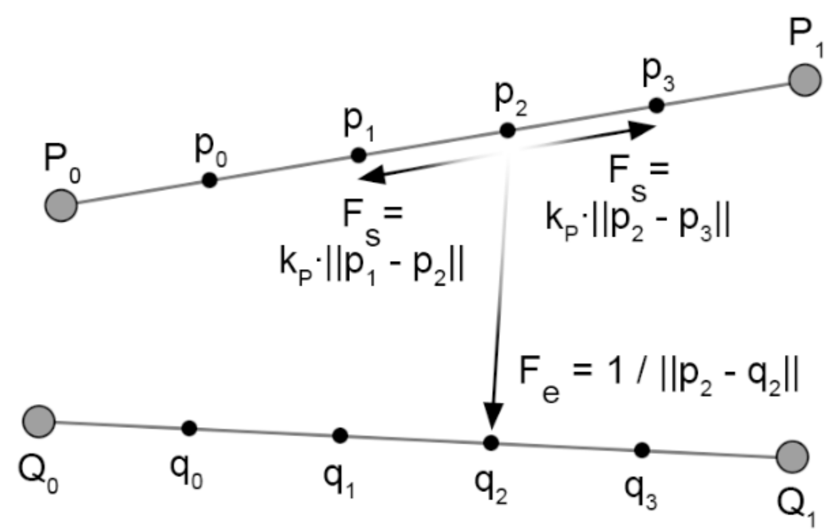

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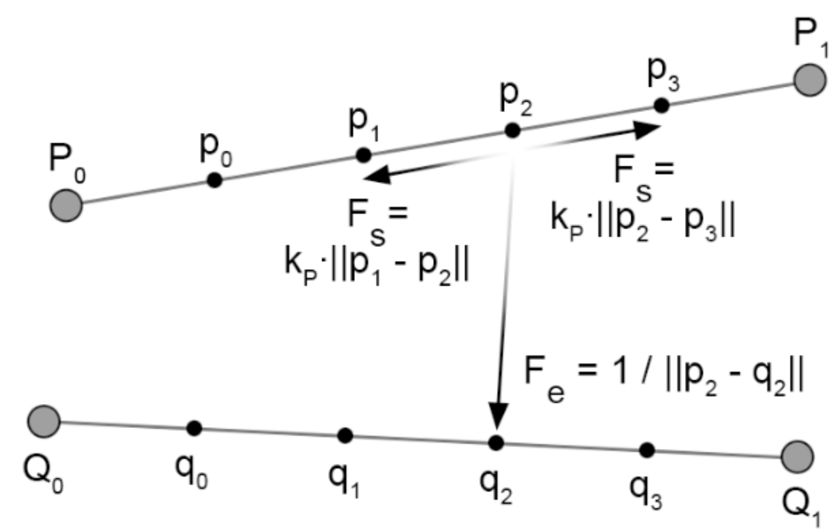
here *K* – global stiffness constant



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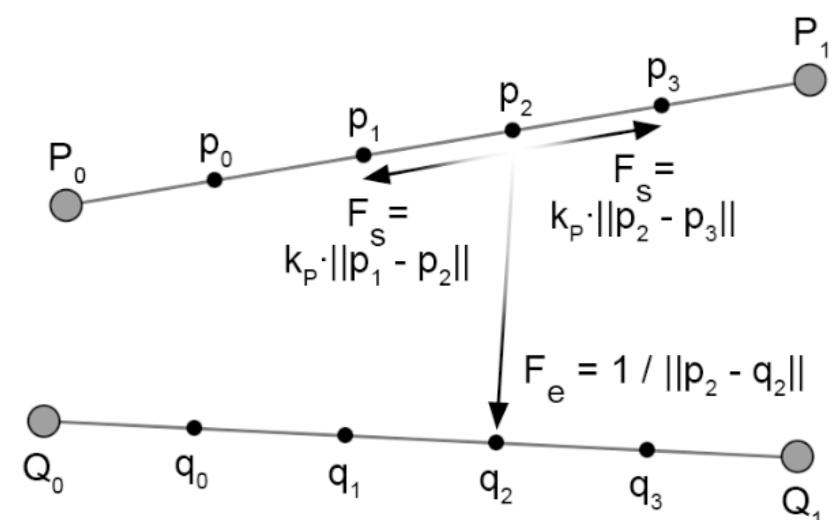
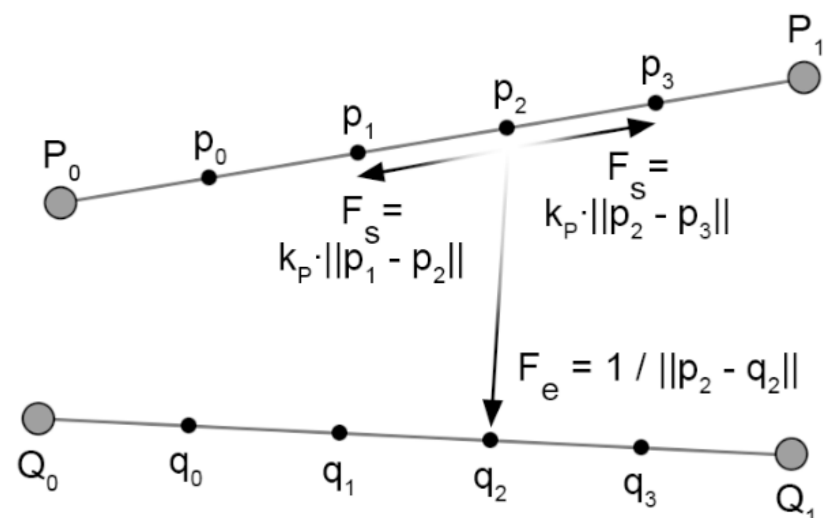
Large values of K make system very stiff



Edge bundling Holten and van Wijk, 2009

- An attraction electrostatic force $F_e(\{p_i, q_i\}) = \frac{1}{||p_i - q_i||}$ is used between each pair of corresponding subdivision points of P and Q, thus between p_0 and q_0, p_1 and q_1, ...



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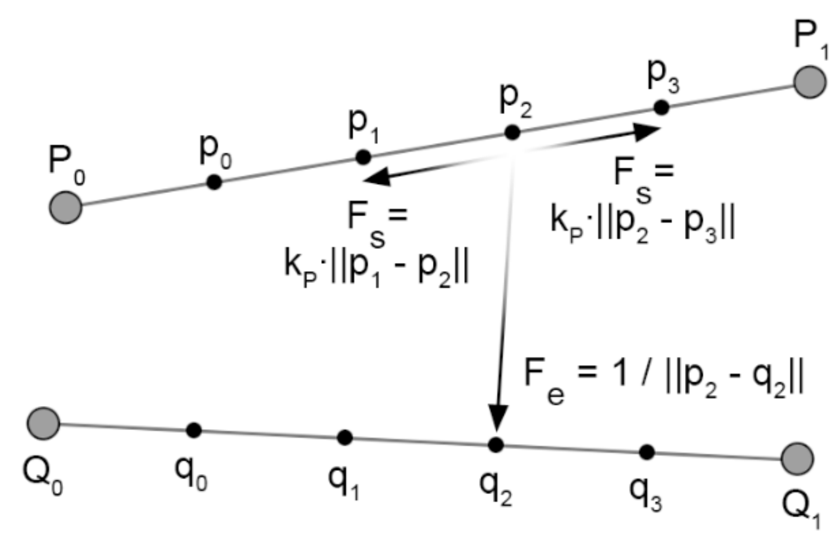

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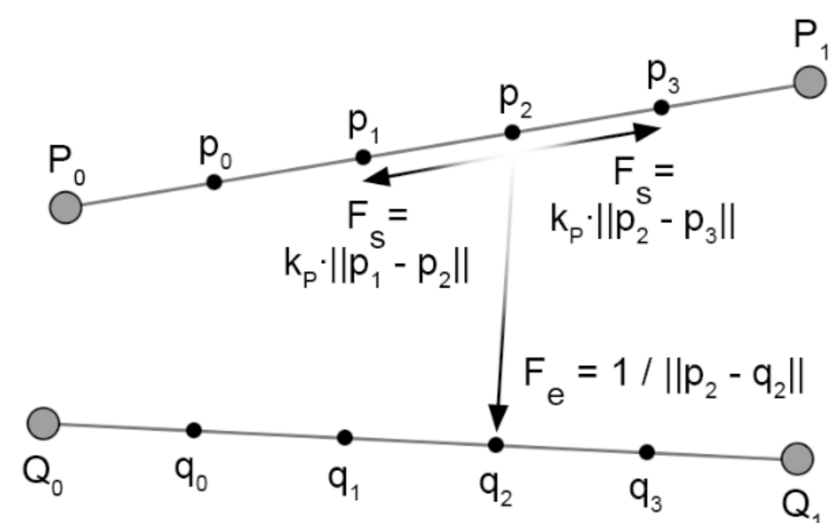

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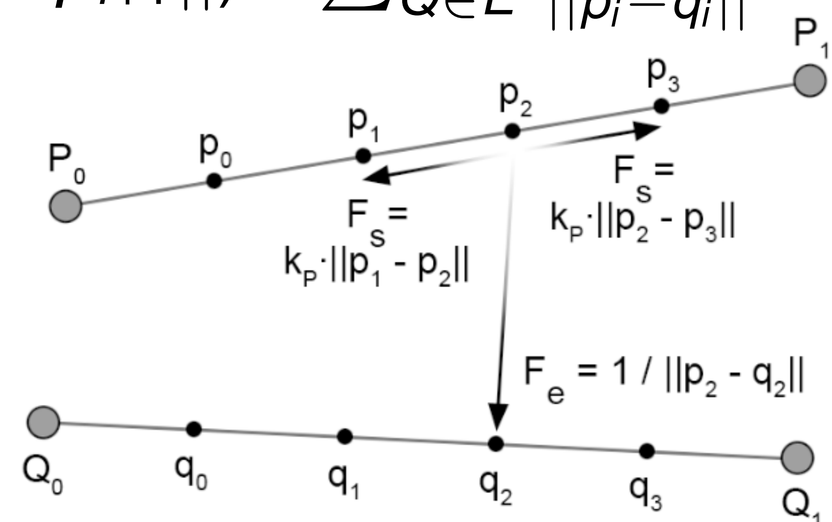

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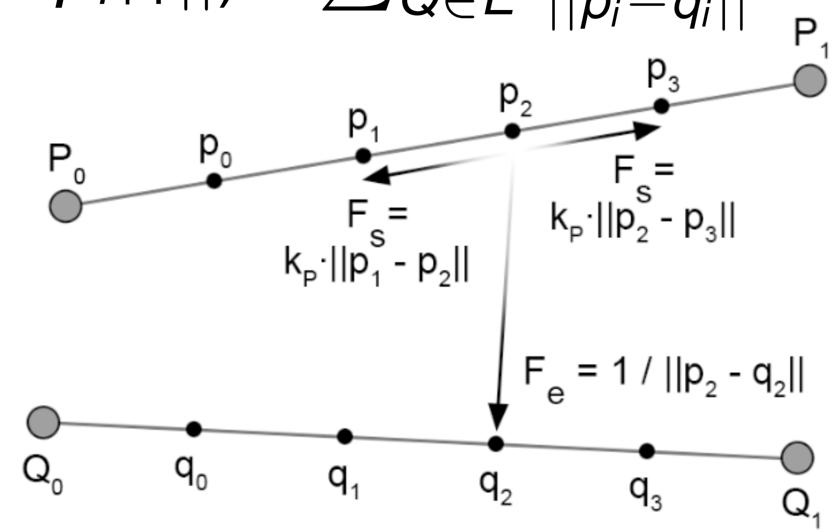
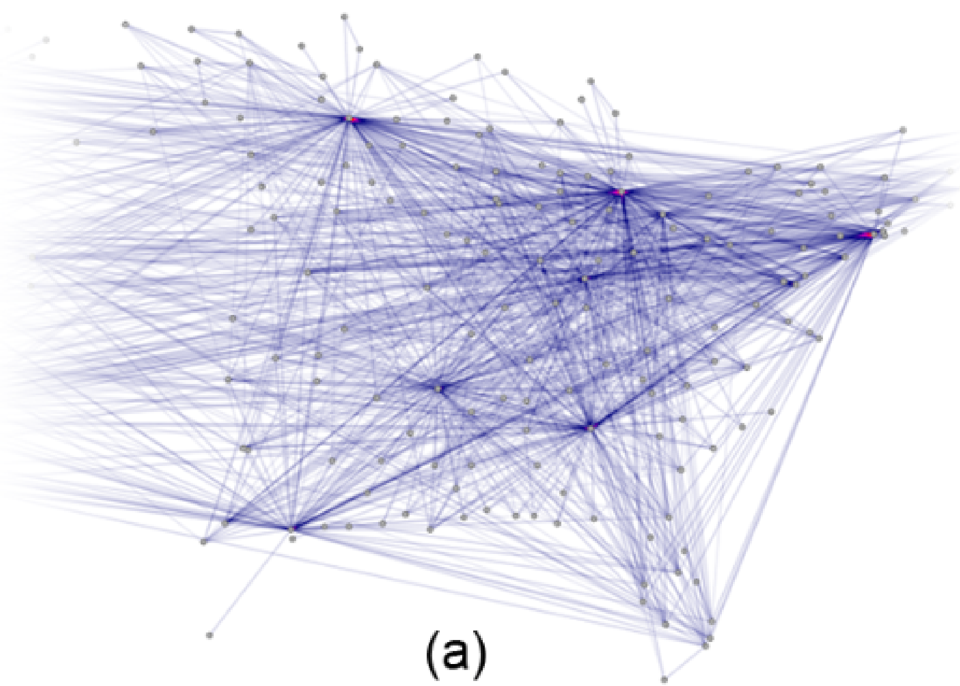
  k_P – constant for edge P

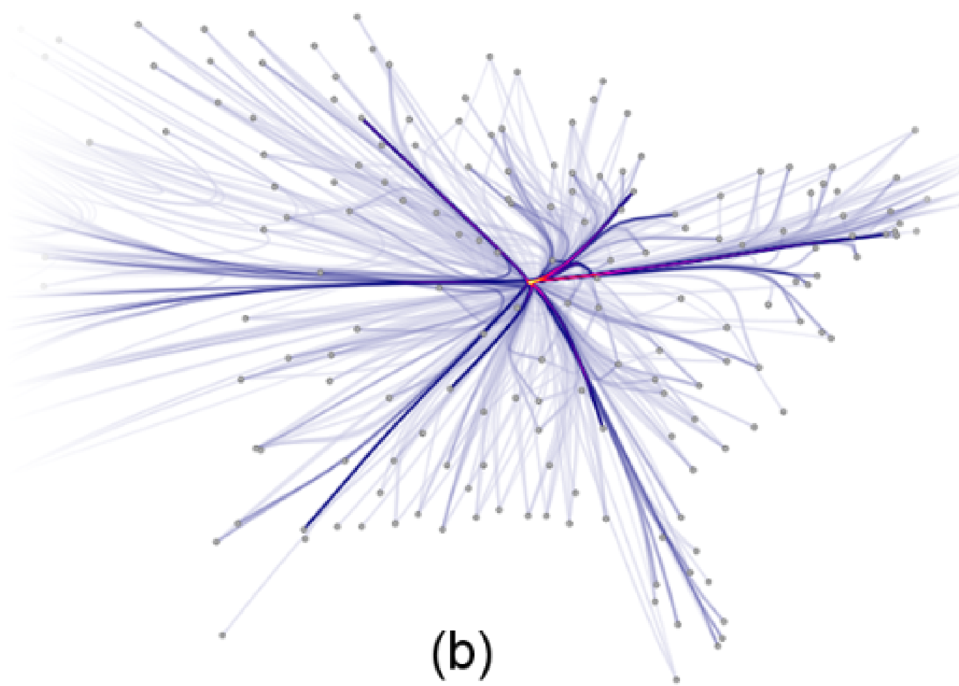


Edge bundling: performance

- Fig.b – performance of the model given up to now. Here all edges interact with all.



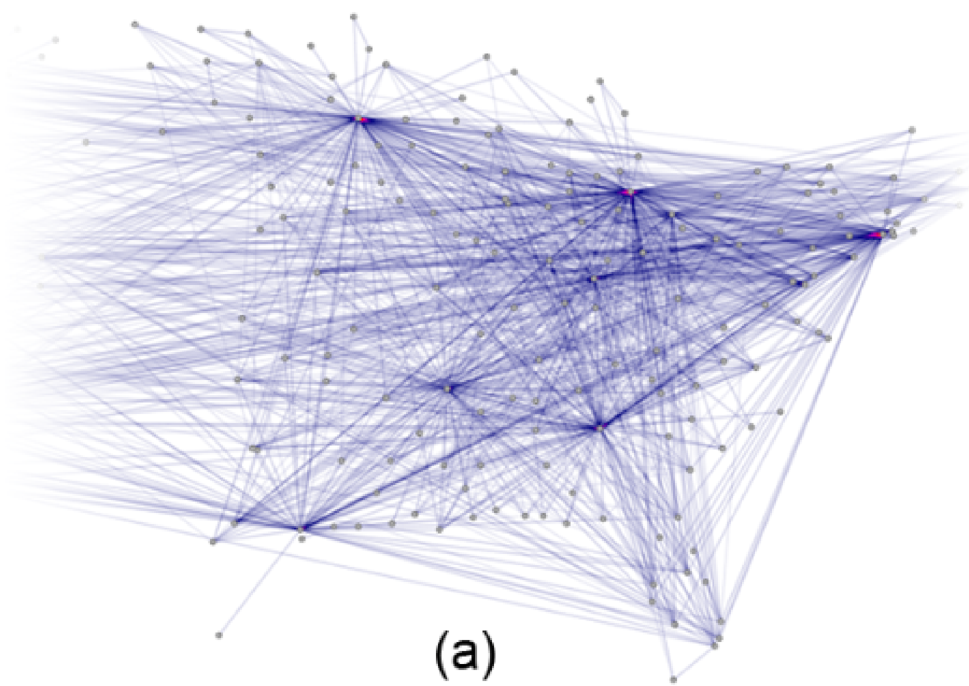
(a)



(b)

Edge bundling: performance

- Fig.b – performance of the model given up to now. Here all edges interact with all.
- Increasing the value of *K* gives less bundling overall and therefore in parts of the graph where a high amount of bundling is still desirable



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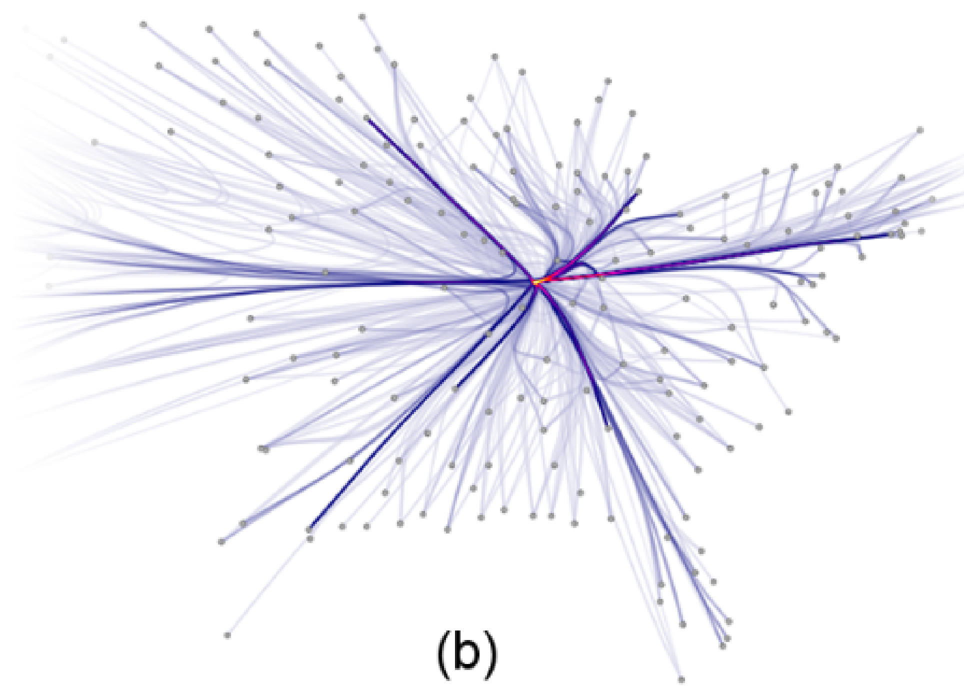
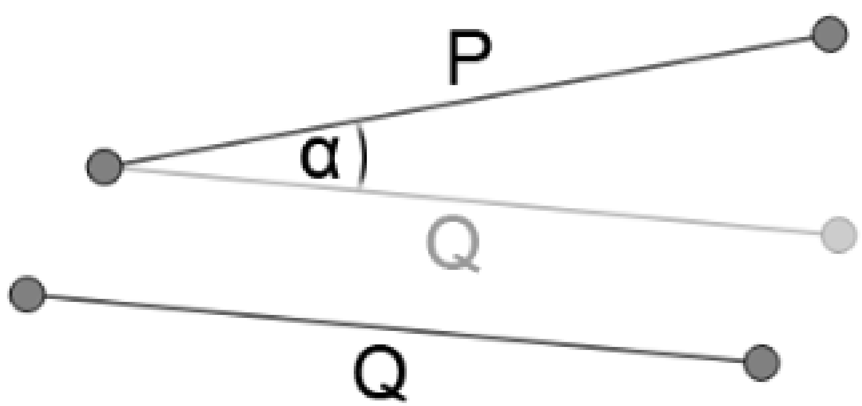


(b)

Edge compatibility measures: angle

- edges that are almost perpendicular should not be bundled together, i.e. should not interact



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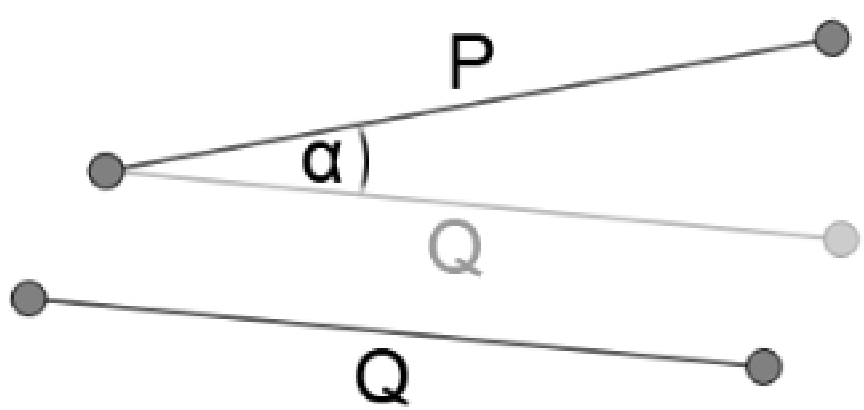
- *angle compatibility* of P and Q is defined as $C_a(P, Q) = |\cos\alpha|$, α is angle between P and Q



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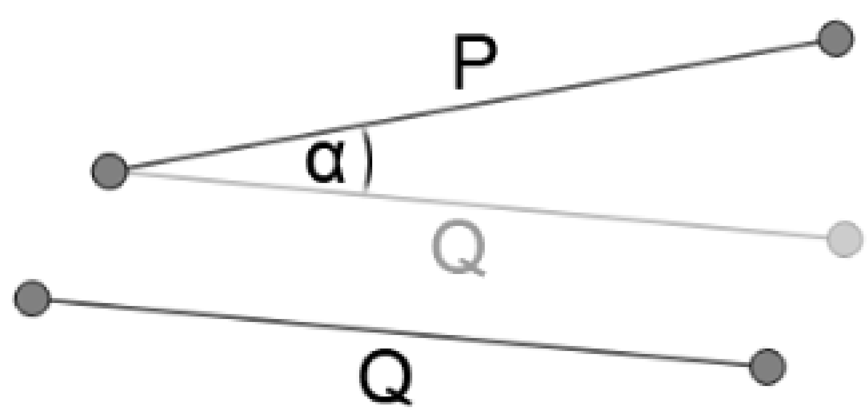

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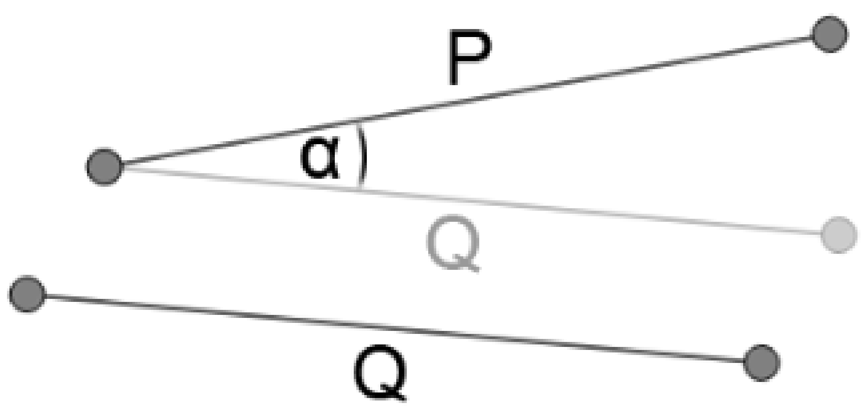

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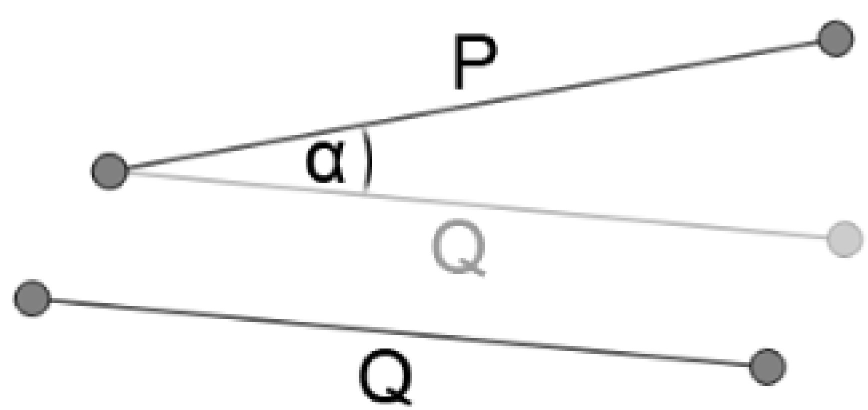

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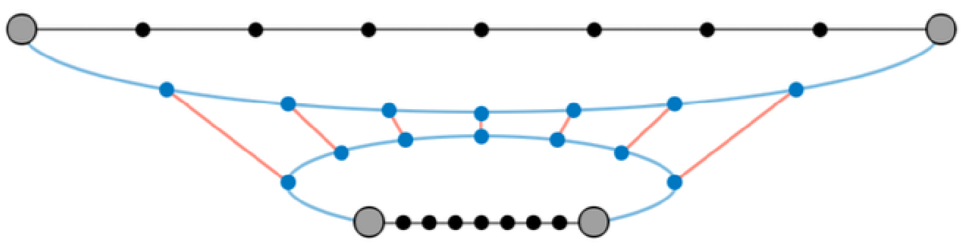

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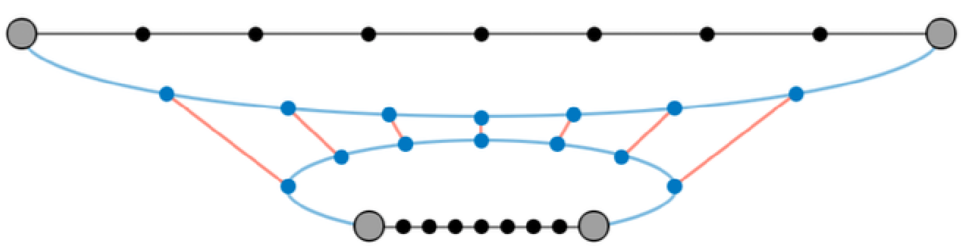

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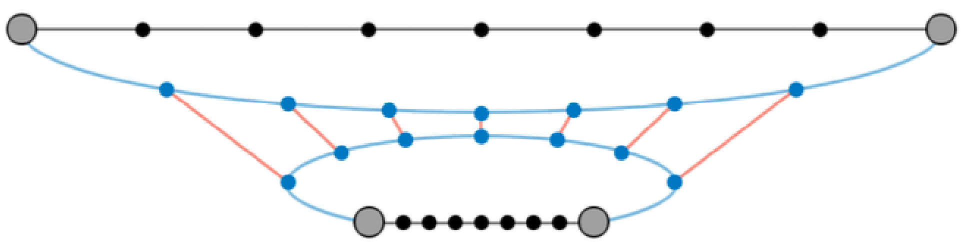

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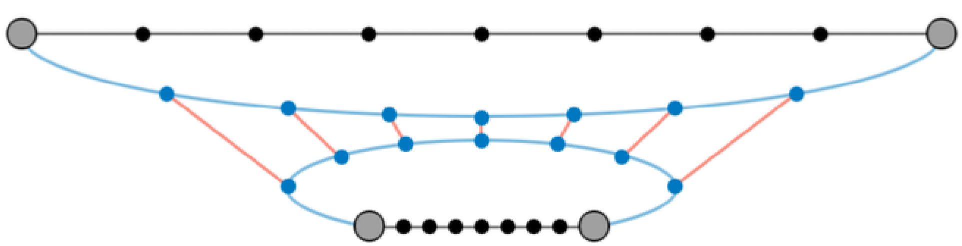

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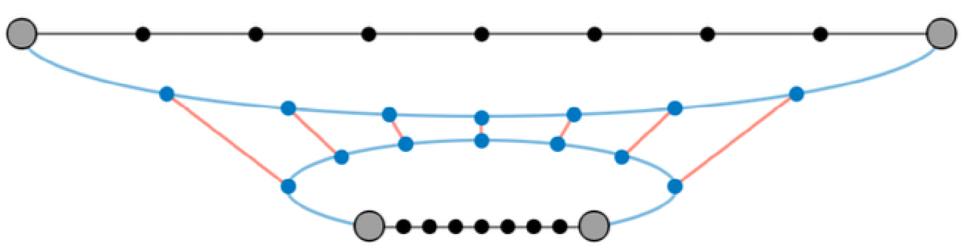

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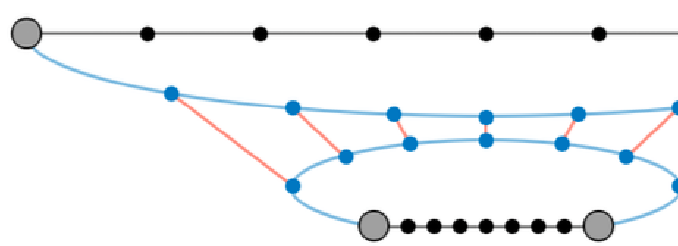
!!!Correction:
Set $|P| = |P| / \min(|P|,|Q|)$ and $|Q| = |Q| / \min(|P|,|Q|)$ - normalize so that shortest has length one, otherwise $C_s(P,Q) > 1$



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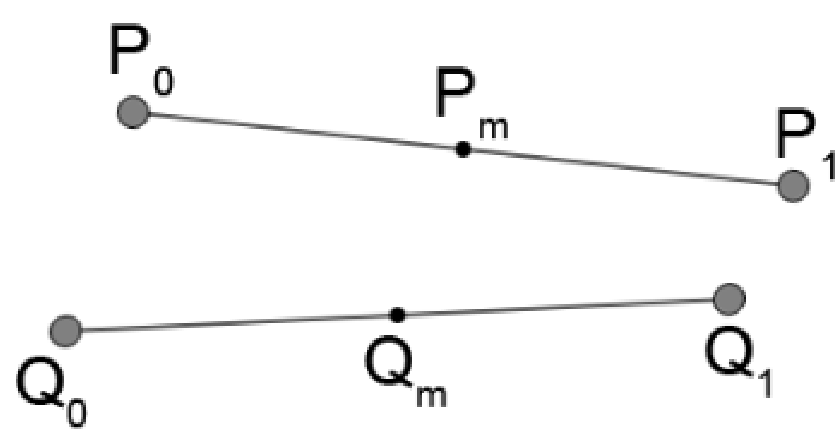

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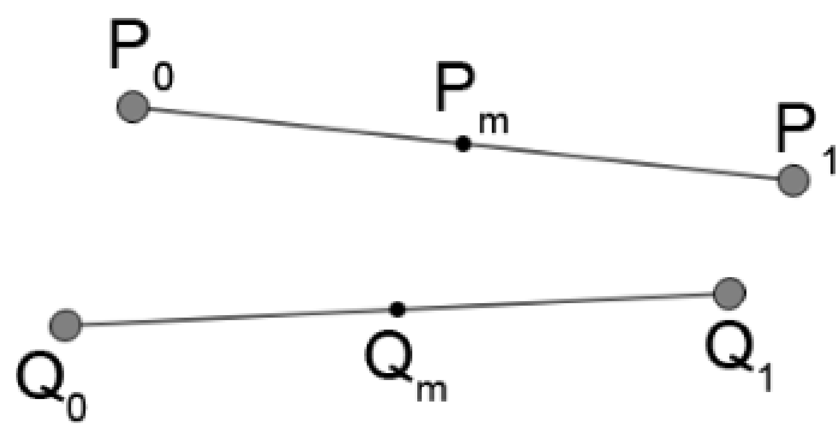

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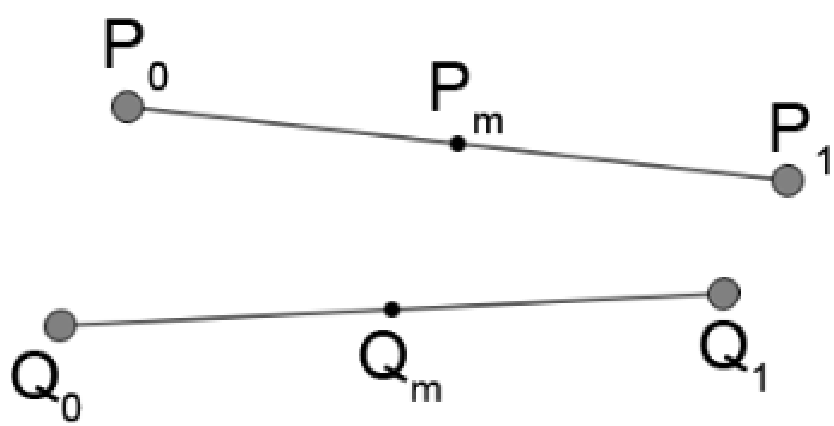
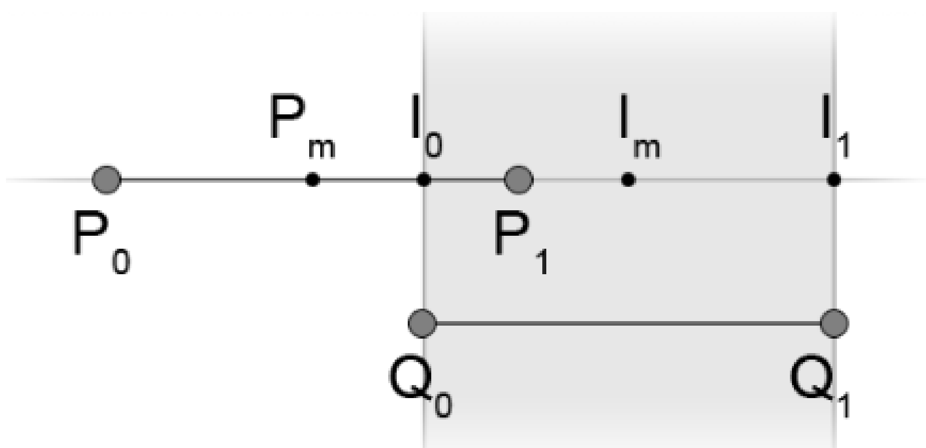


Edge compatibility measures: visibility

- it is possible that edges are parallel, equal in length, and close together, but should nevertheless have a low bundling



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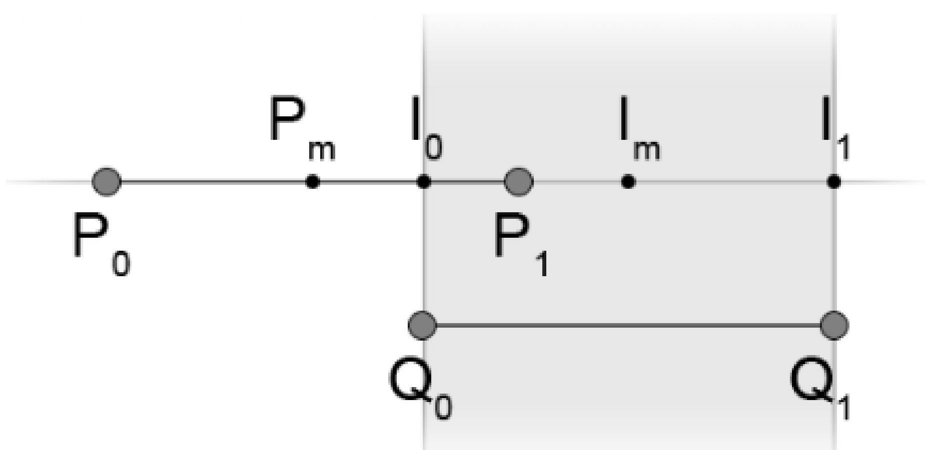

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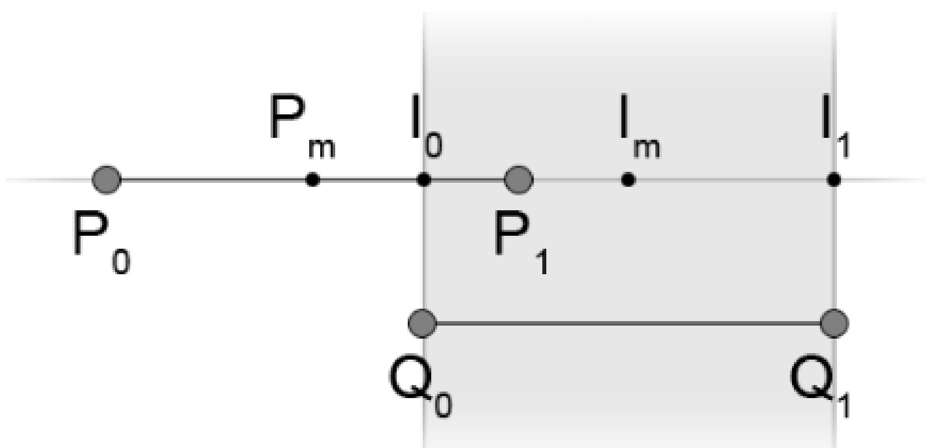

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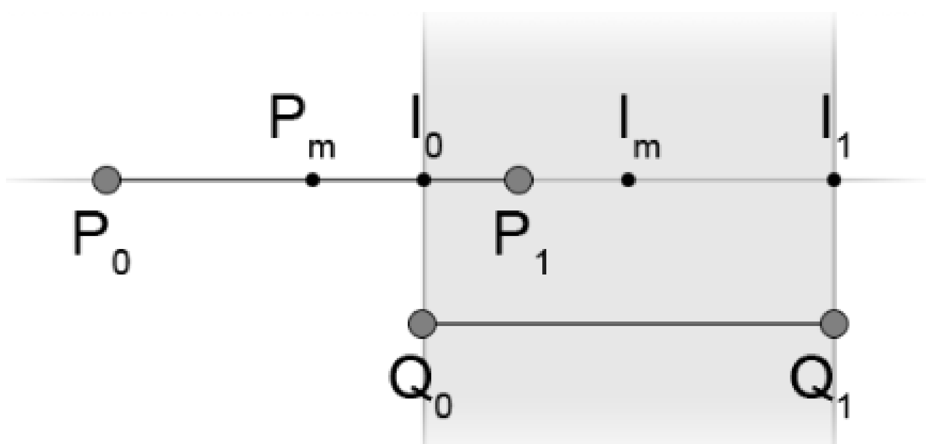

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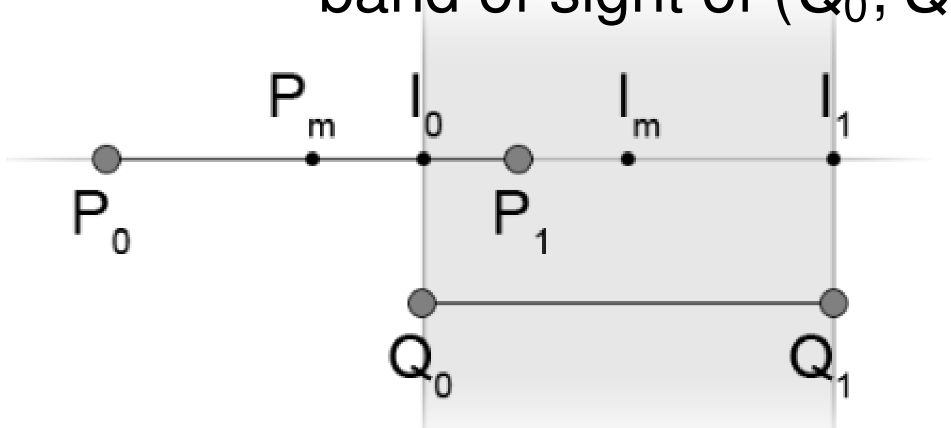
band of sight of (Q_0, Q_1)



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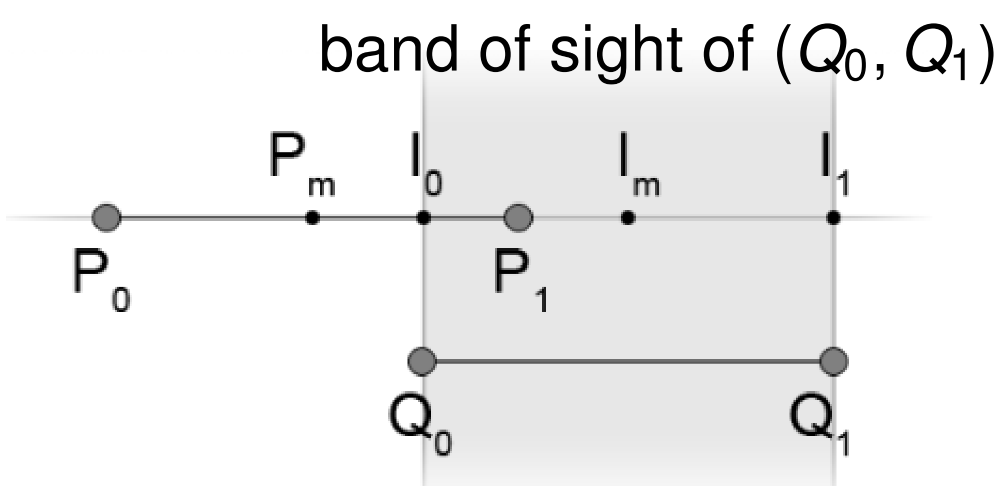

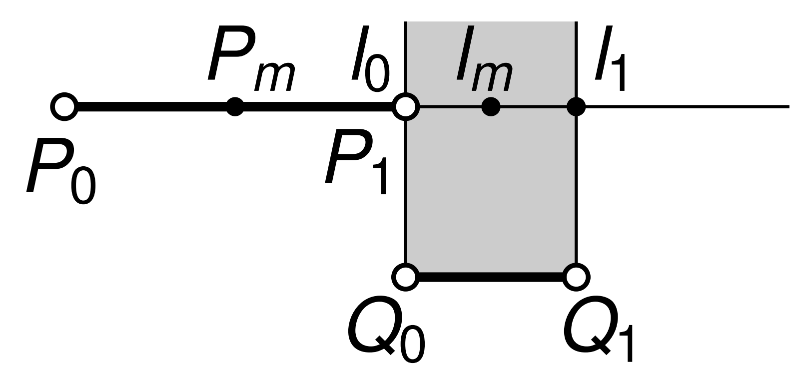

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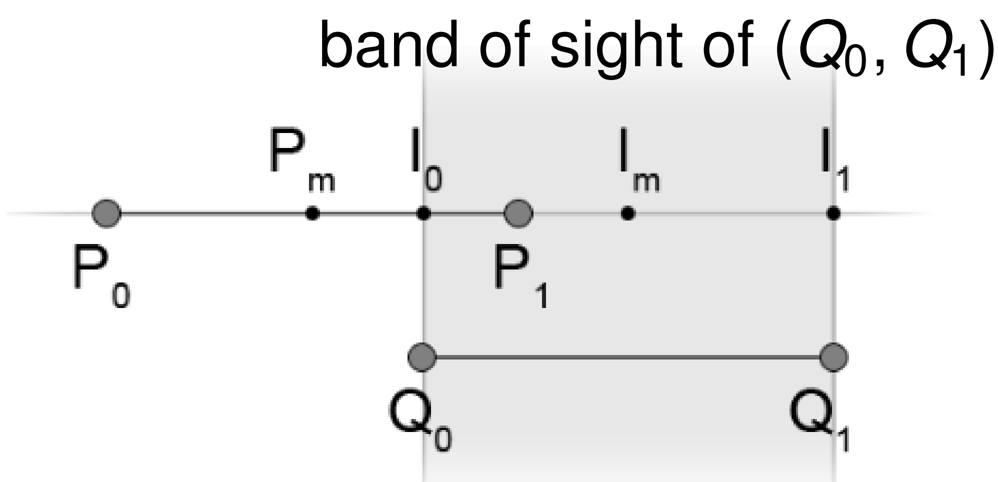

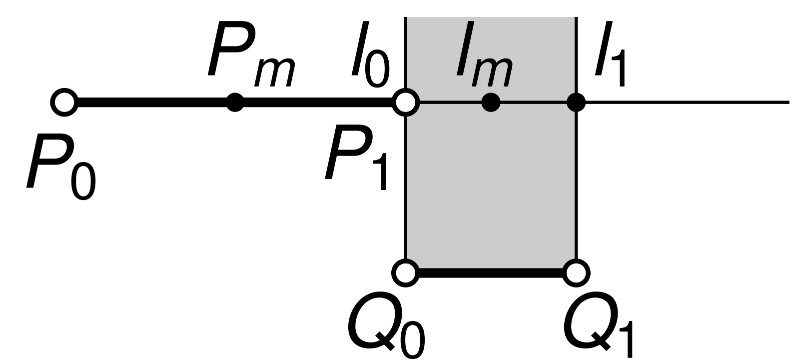

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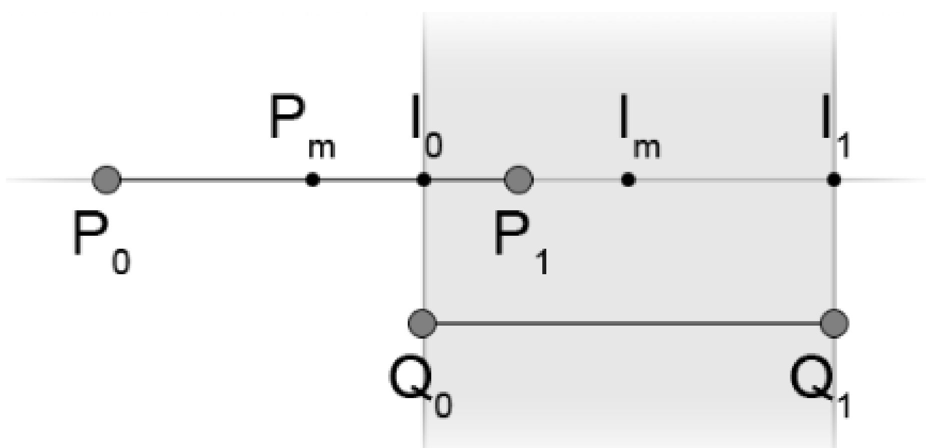
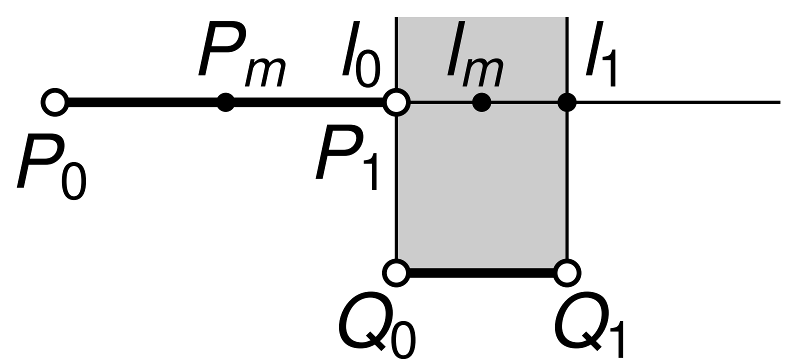
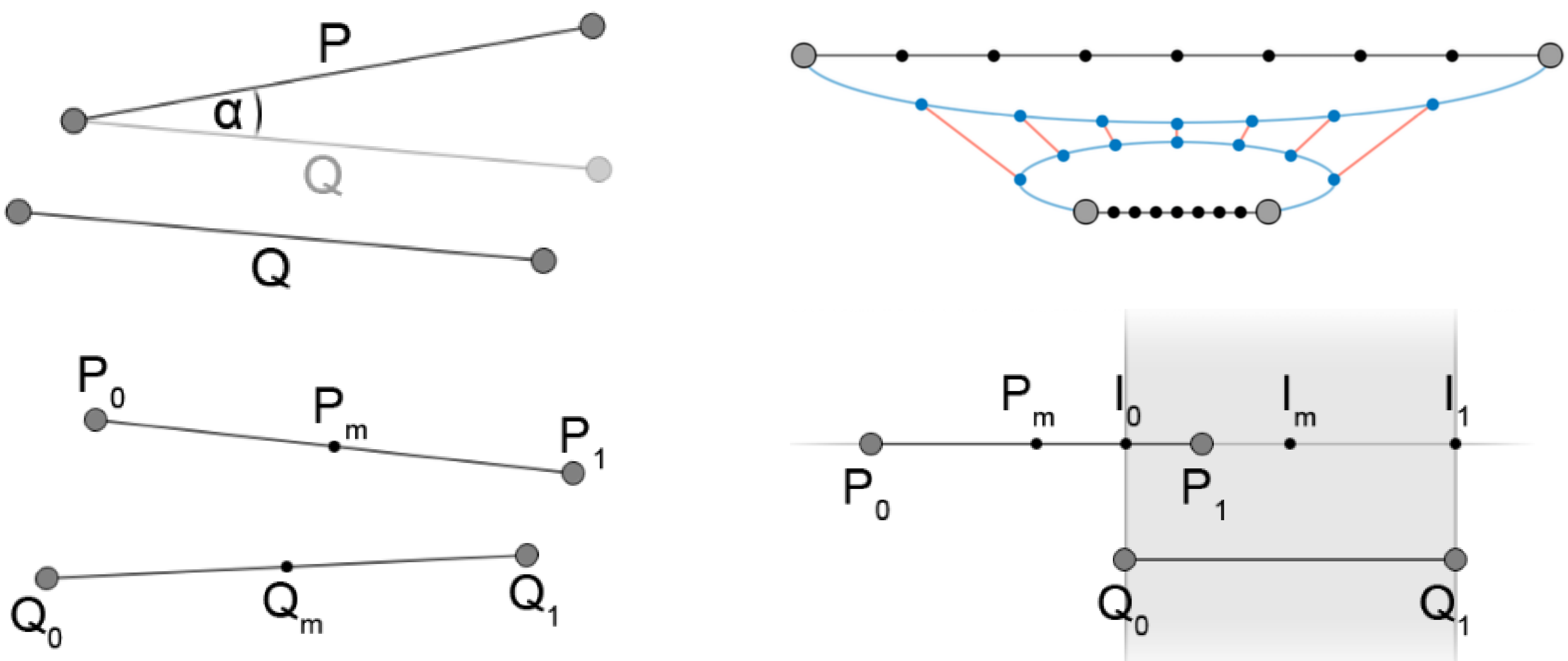




Edge compatibility measures: combined

- The overall compatibility is defined as
  $C_e(P, Q) = C_a(P, Q) \cdot C_s(P, Q) \cdot C_p(P, Q) \cdot C_v(P, Q)$



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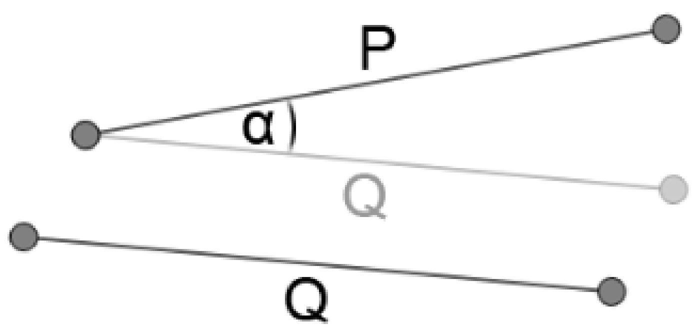
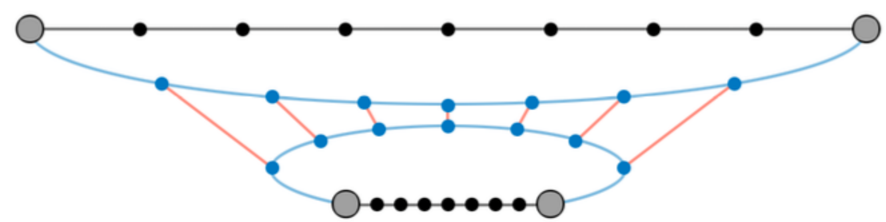
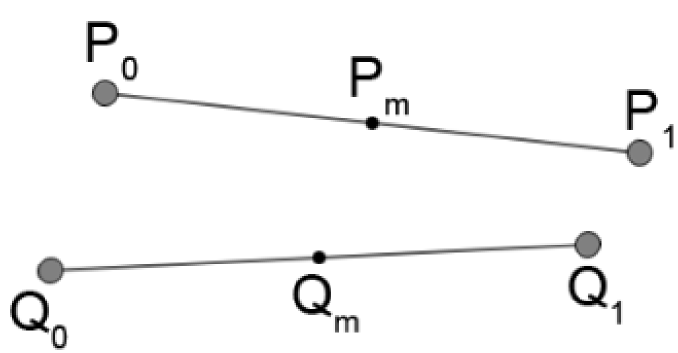
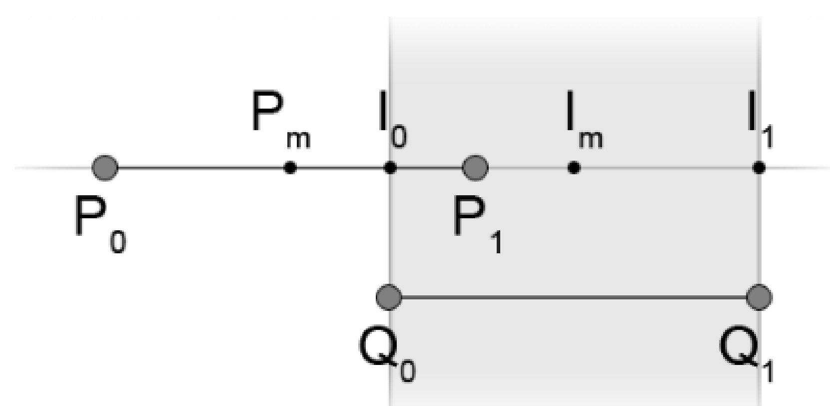
- The overall force on point p_i is then redefined as
  $F_{p_i} = k_P \cdot (||p_{i-1} - p_i)|| + ||p_i - p_{i+1}||) + \sum_{Q \in E} \frac{C_e(P,Q)}{||p_i - q_i||}$
  k_P – constant for edge P









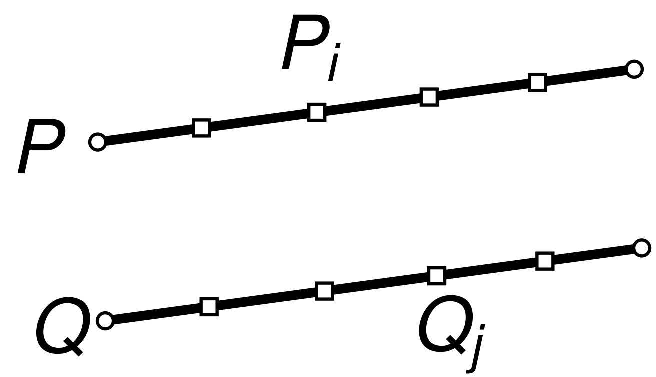
Edge bundling summary

Input: $G = (V, E)$ undirected graph with vertex placement,
number of cycles $C \in \mathbb{N}$,
number of iterations in the first cycle $I_0 \in \mathbb{N}$,
step size $s_0 \in \mathbb{N}$,
number of subdivision points in the first cycle n_0
interaction function $C_e : E \times E \to \mathbb{R}$
Output: Layout with bundled edges

$n \leftarrow n_0$ initial number of subdivisions
$t \leftarrow 1$ iteration counter
$I \leftarrow I_0$ number of iterations in the first cycle
$c \leftarrow 1$ cycle counter
$s \leftarrow s_0$ step size

Edge bundling summary

while $c < C$ **do**
- **foreach** $P \in E$ **do**
  - subdivide P by n points $P_1 \ldots P_n$; $B \leftarrow B \cup \bigcup_{P \in E} \{P_1 \ldots P_n\}$
- **foreach** $P \in E$ **do**
  - **foreach** $0 < i < n$ **do**
    - $F_{P_i} = k_P \cdot (||P_{i-1} - P_i)|| + ||P_i - P_{i+1}||)$
    - **foreach** $Q \neq P \in E$ **do**
      - **foreach** $0 < j < n$ **do**
        - $F_{P_i} = F_{P_i} + \frac{C_e(P,Q)}{||P_i - Q_j||}$;
- **foreach** $p \in B$ **do**
  - $p \leftarrow p + s \cdot F_p$
- $t \leftarrow t + 1$
- **if** $t == I$ **then**
  - $t \leftarrow 1$; $c \leftarrow c + 1$; $n \leftarrow 2n$;
  - $s \leftarrow s/2$; decrease(I);



Edge bundling: experiments

- experimentally the following values are suggested $n_0 = 1$, $s_0 = 0.04$, $C = 6$, and $l_0 = 50$.

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cycle	0	1	2	3	4	5
n	1	2	4	8	16	32
s	.04	.02	.01	.005	.0025	.00125
I	50	33	22	15	9	7

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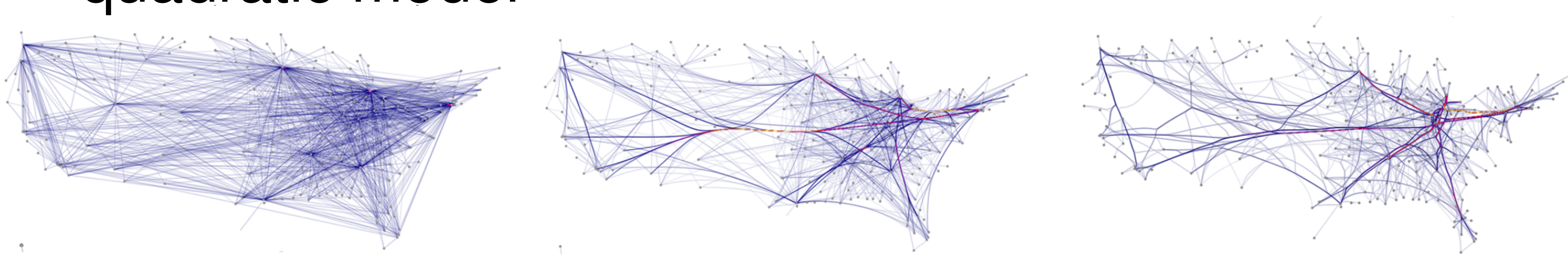
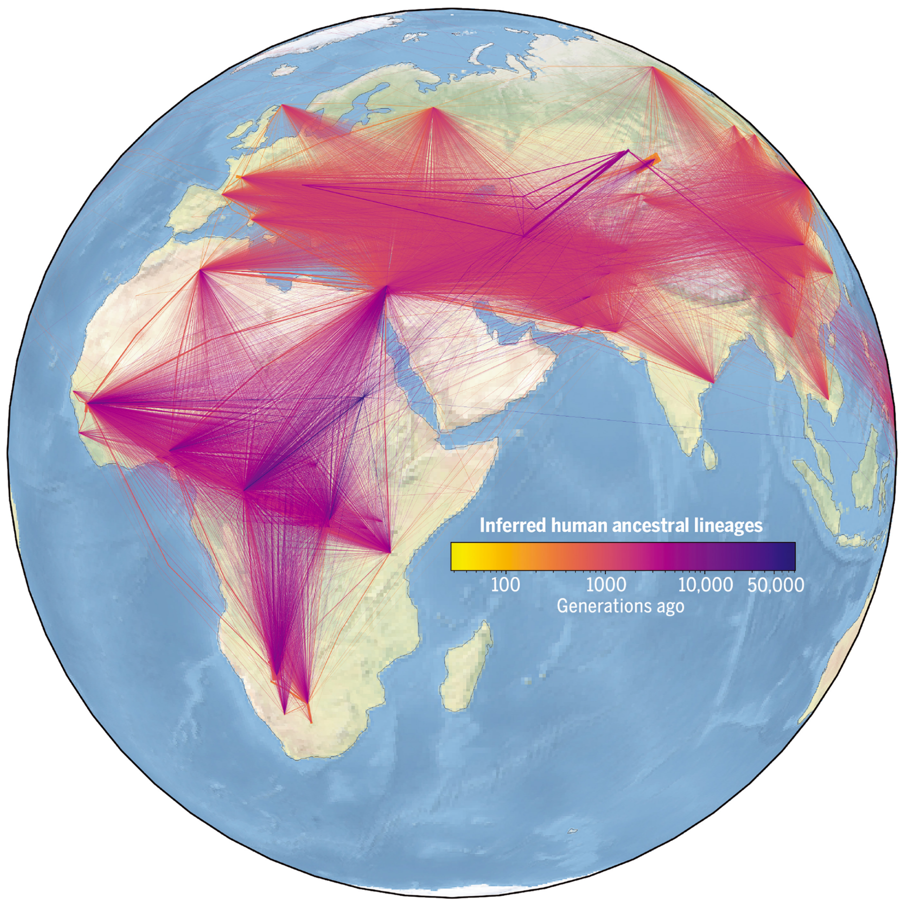
- US airlines graph with inverse linear and inverse quadratic model



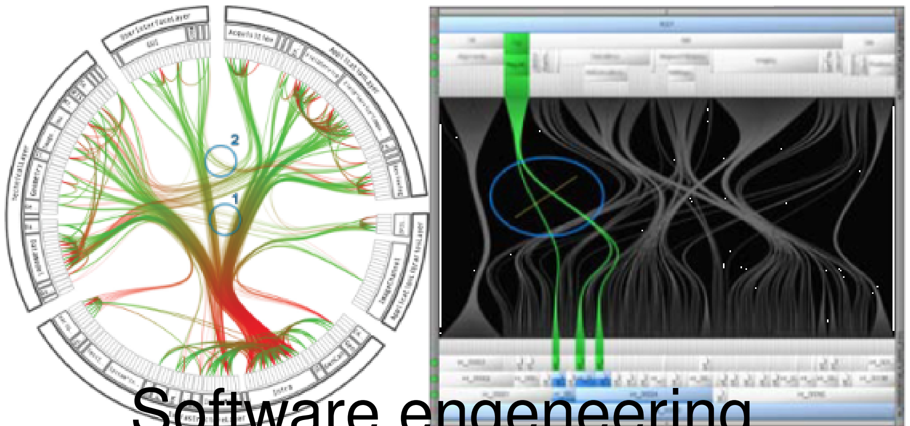
Edge bundling: inspiration

Inspiration: edges are ancestor-descendant relationship in the genealogy of modern and ancient genomes. Edge width – how many times the relationship is observed, color – age of the ancestor



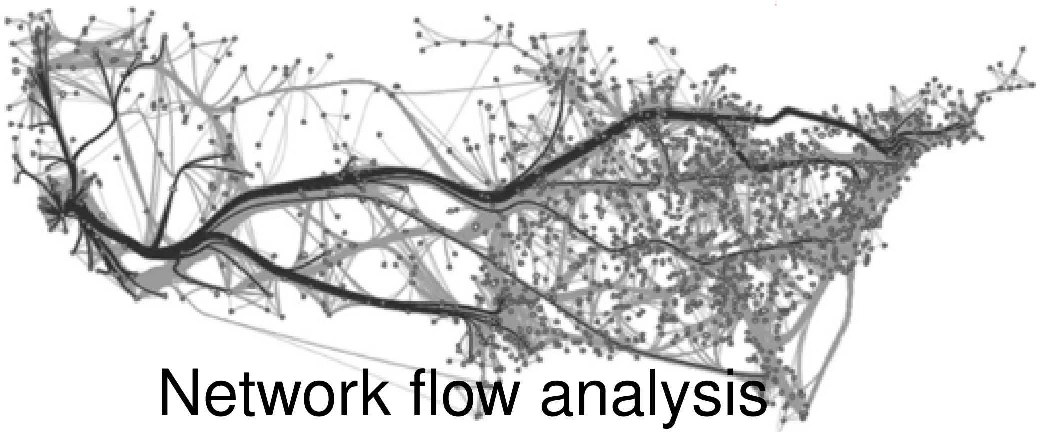
A unified genealogy of modern and ancient genomes, Wohns et al. Nature 2022

Edge bundling: discussion

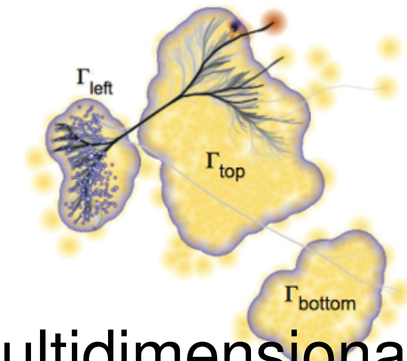


Software engeneering

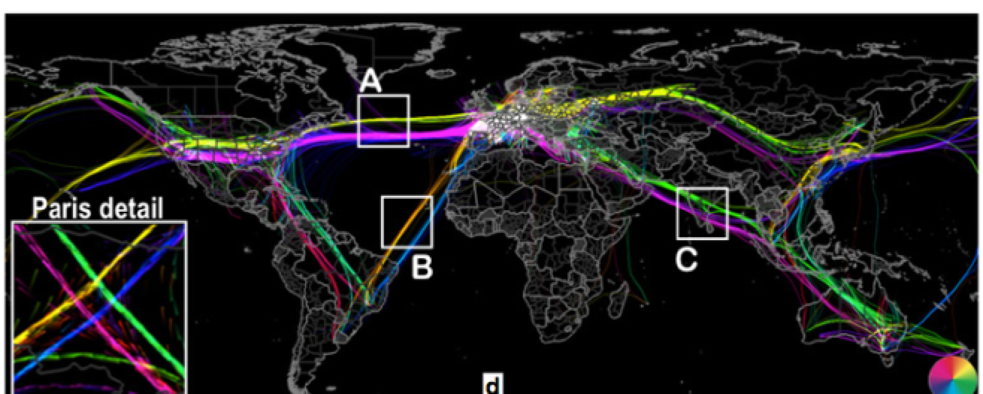
Bioinformatics



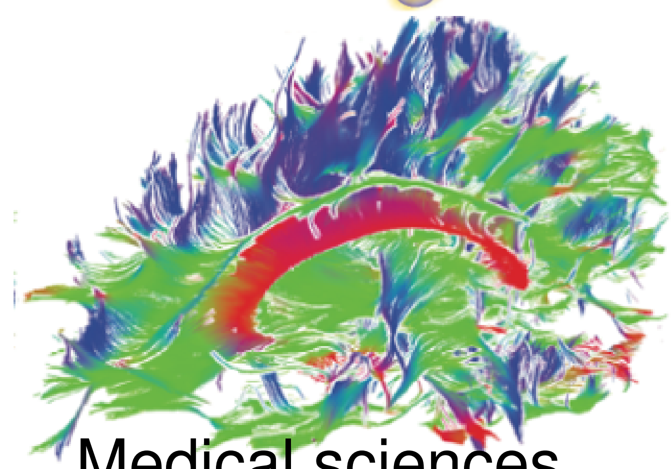
Network flow analysis



Multidimensional data

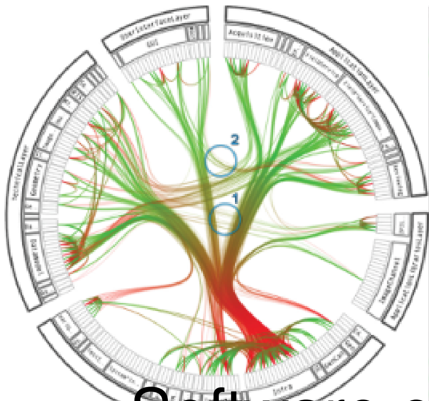


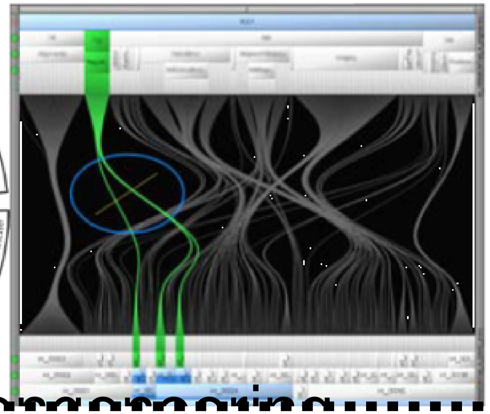
Air traffic control

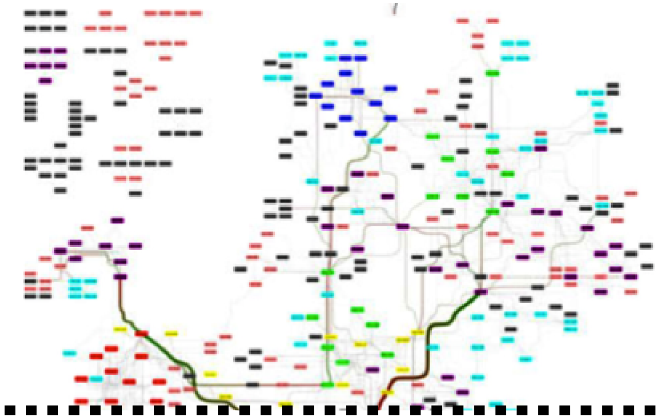


Medical sciences

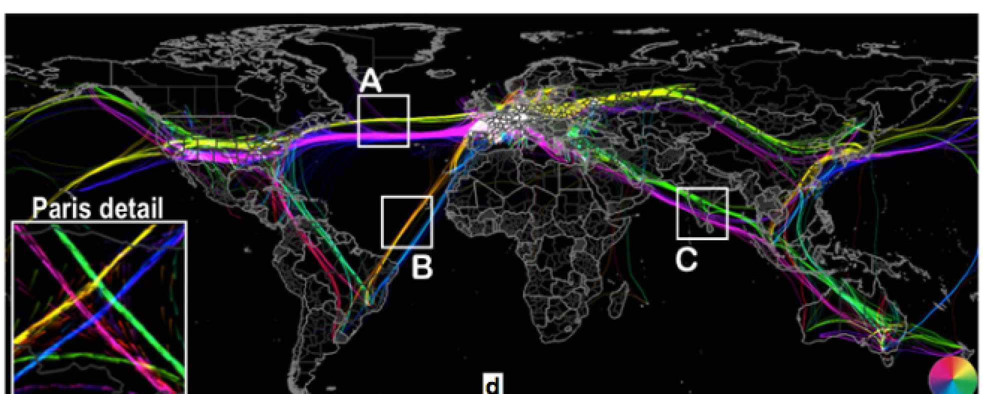
Edge bundling: discussion



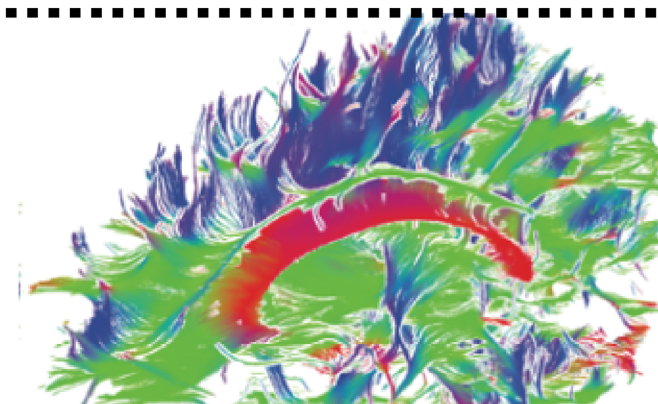




- What are the benefits and the drawbacks of the bundled layouts?
- When are the edge bundling techniques appropriate to use?



Air traffic control



Medical sciences

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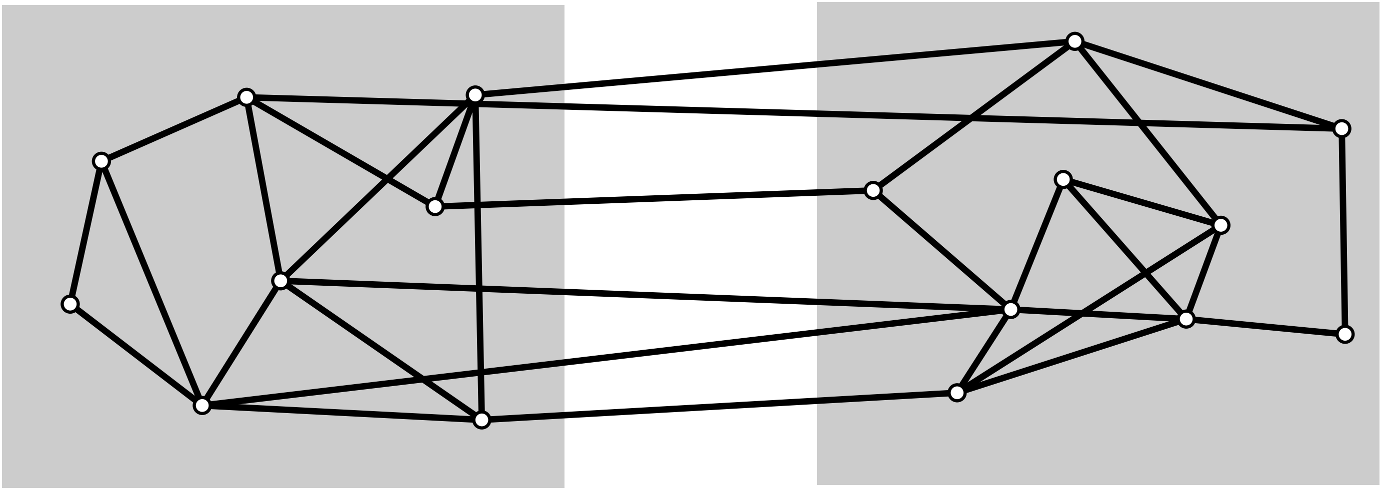

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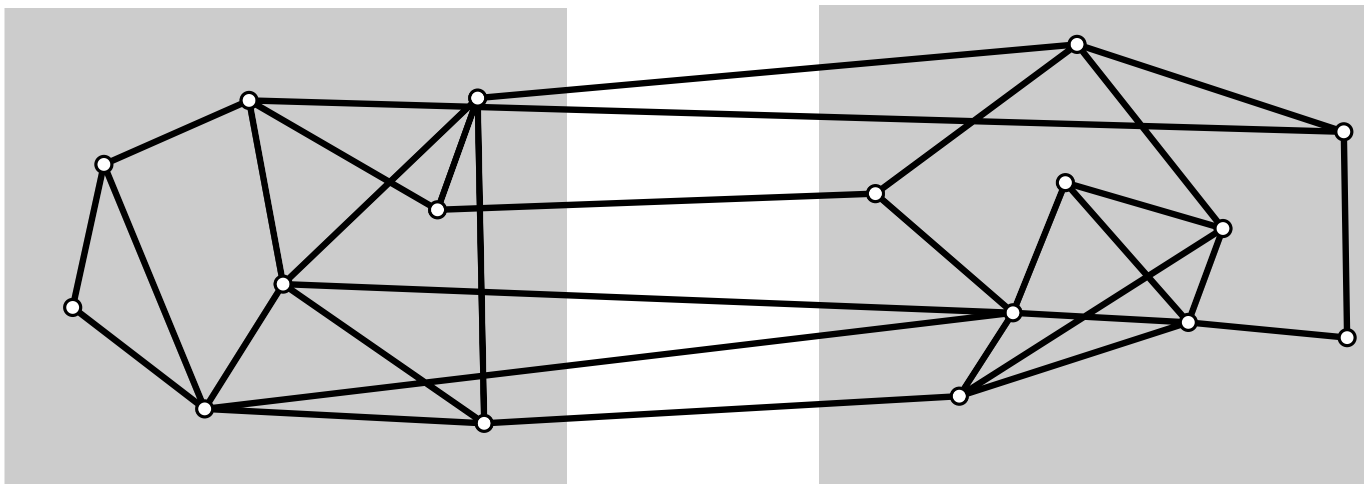

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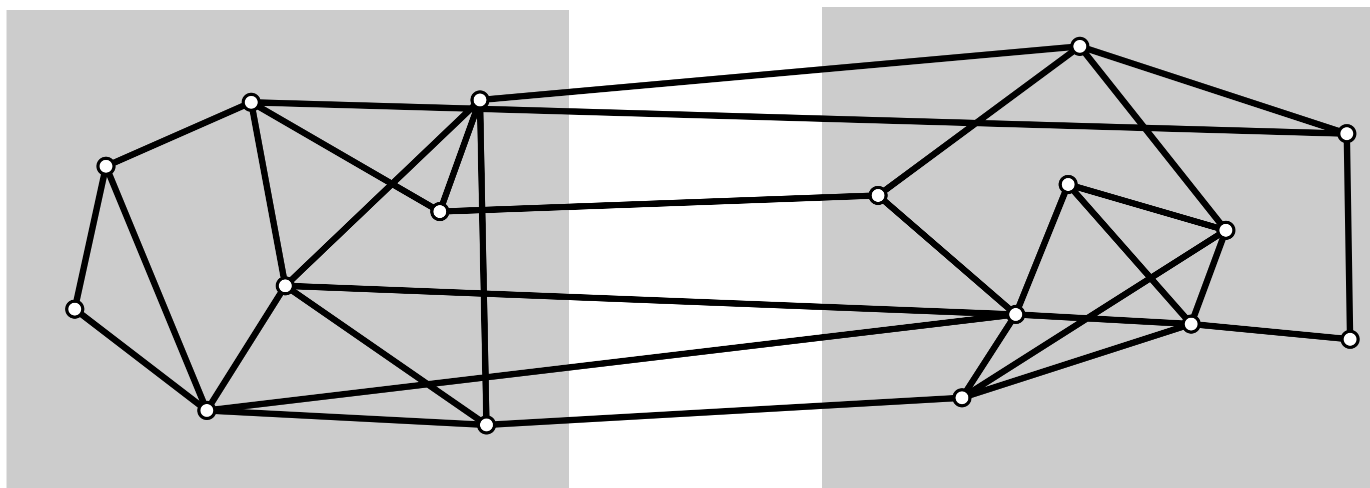

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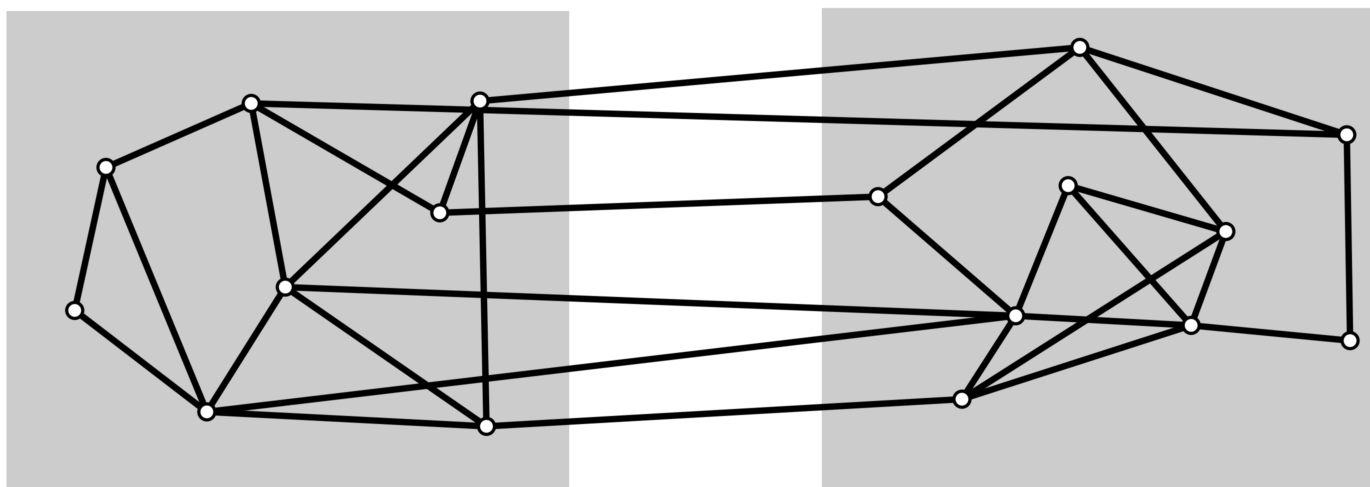

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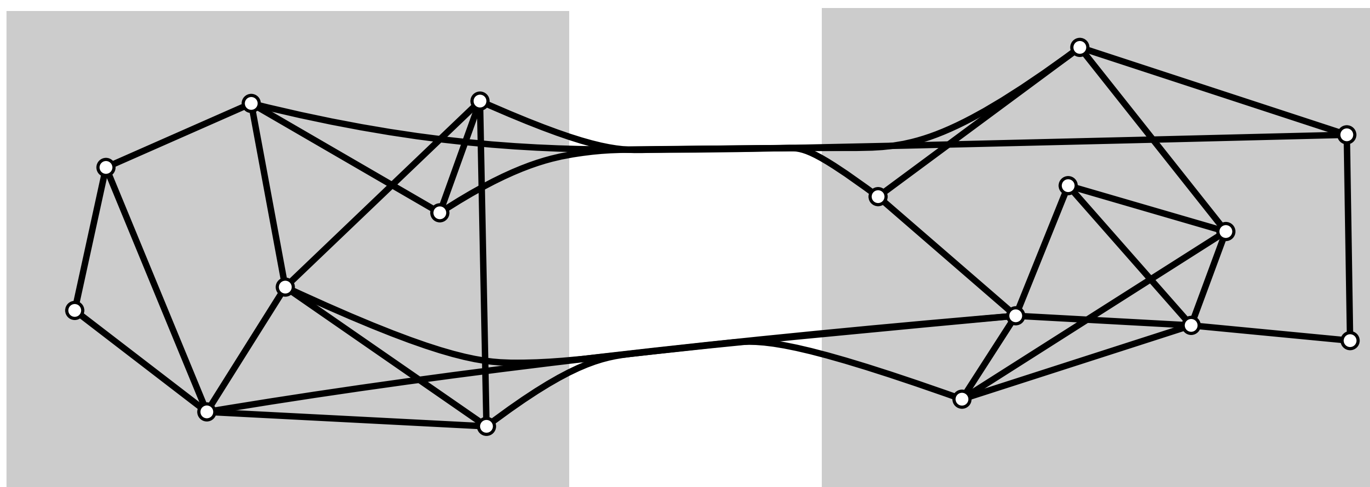

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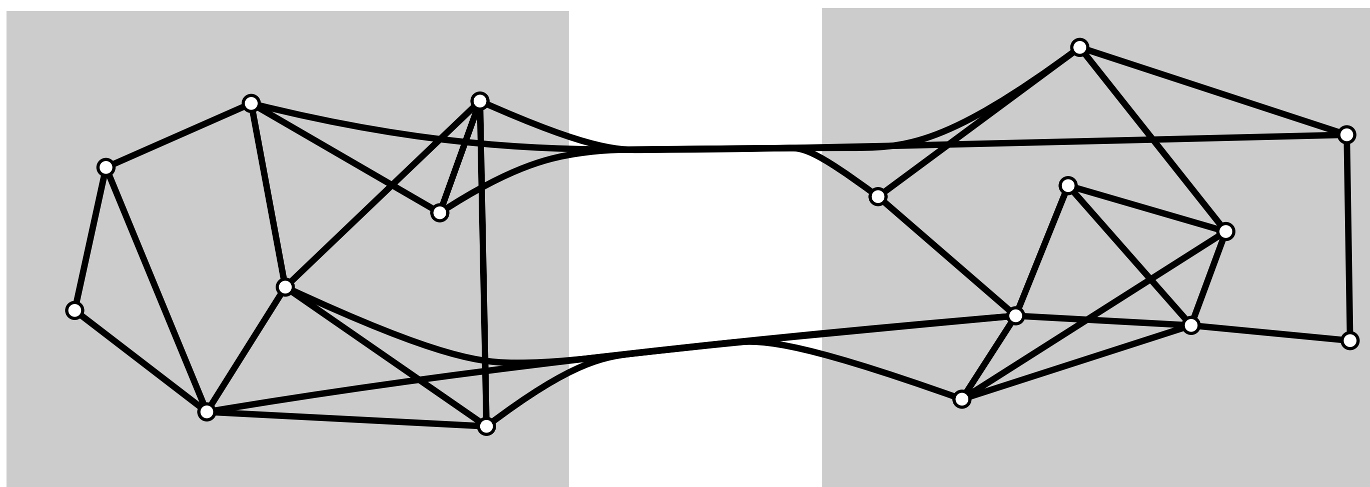

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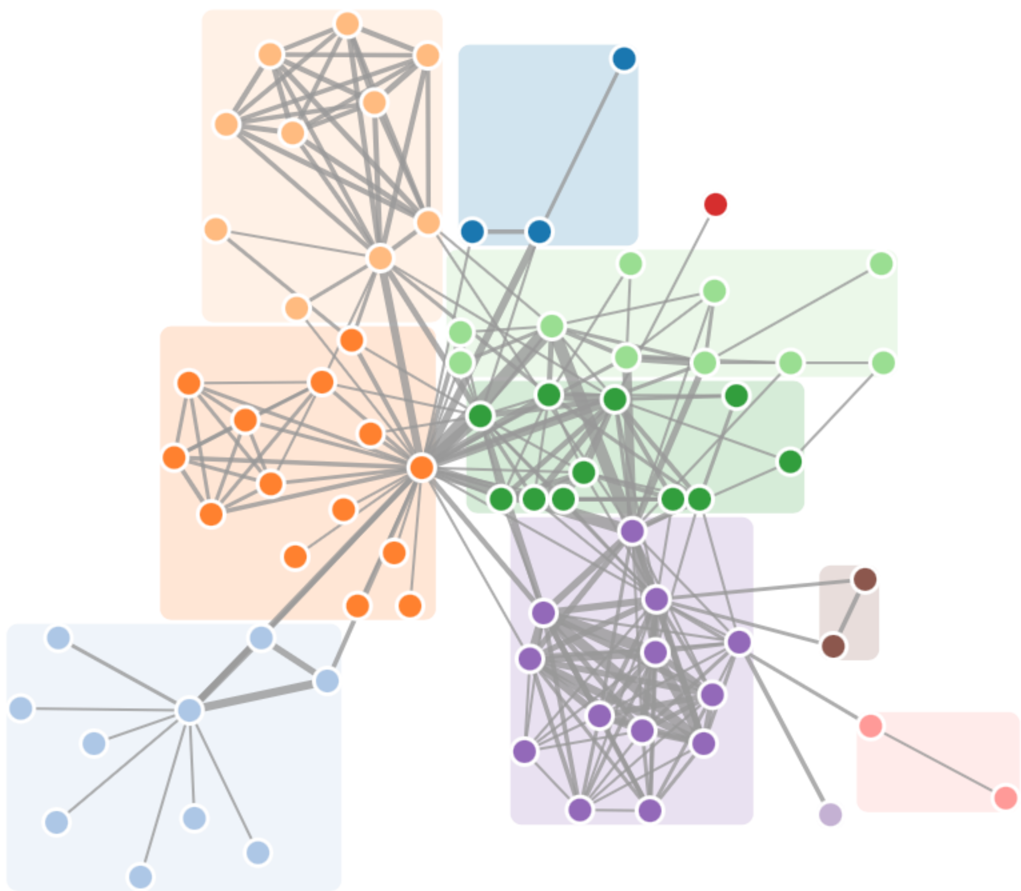
- implement the bundling algorithm and apply it to inter-cluster (inter-layer) edges
- experiment with political blogosphere, argument network (besides the two clusters, nodes and edges have different types)

Tutorial task (bonus)

- expand your method to work for many layers/clusters
- you need to find a way to arrange an arbitrary number of boxes – inspiration `cola.js`, `yEd`



Reading and Next

Additional Reading

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Next

	March 6	-	Visualization of multilevel networks	Tamara	
11	March 11	-	Tutorial: Step 5	Alister	Step 4
	March 13	-	High-dimensional data visualization	Alex	
12	March 18	-	Tutorial: Step 6	Alister	Step 5
	March 20	-	High-dimensional data visualization: advanced	Alex	
13	March 25	-	Tutorial: Step 7	Alister	Step 6
	March 27	-	—		
14	April 2	17:15-19:00	Final Presentations	Students	Step 7
	April 3	9:00-12:45	Final Presentations	Students	
	April 8				Final deliverables

Final Remarks

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